The Architecture and Mechanics of Elliptical Domes

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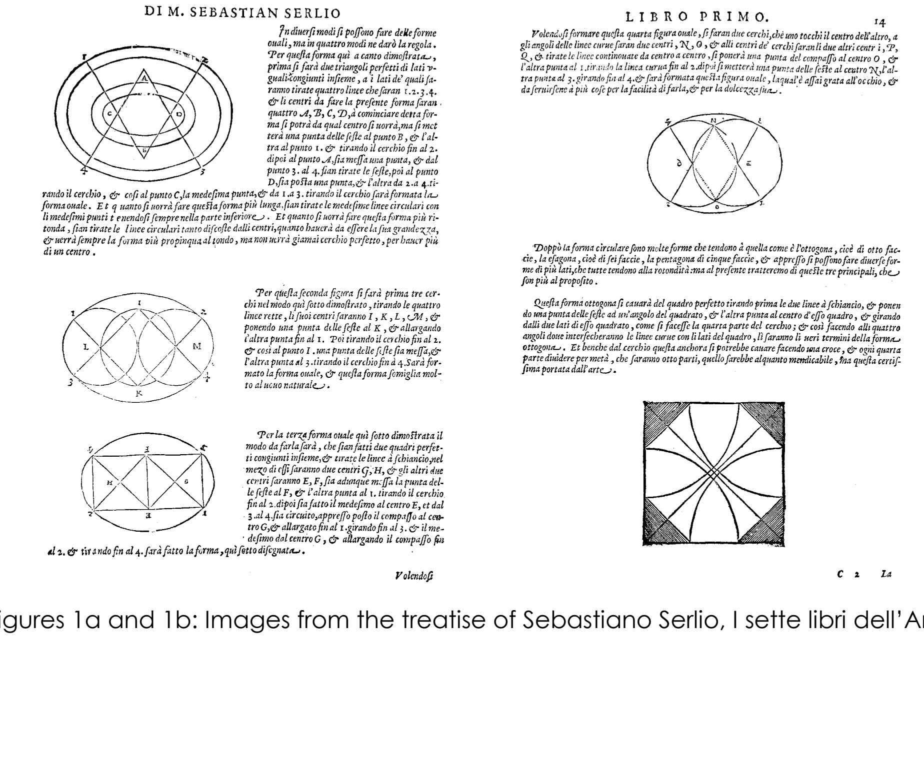
ABSTRACT: This paper focuses on some aspects of a particular kind of structure, the elliptical dome, a structural typology that was the object of study by the great architects from the time of Sebastiano Serlio all the way through the Baroque age. It was well suited for roofing architectural spaces, above all in religious buildings which, built in the full flower of the Counter-Reformation, in XVII Century, were theatrical and evocative of the newly-reaffirmed Catholic Church. This present study illustrates the masterly structures of Rome, then moves to the areas of Piedmont and Liguria, as a necessary preamble to the study of the static and kinematic behaviour of these special structures, characterised by two axes of symmetry defining a bi-axially symmetric space, characteristic of buildings with a centralised plan with hierarchical axes. In particular, this paper includes the study carried out on the dome of the church of San Giuseppe in Voghera (province of Pavia), built in XVIII Century, to determine the limit conditions of equilibrium.

SHAPE AND GEOMETRY

The geometric construction of a dome with an elliptical plan consists in laying out two equilateral triangles with the same base, thus forming a rhombus. The axes of the four chapels that open towards the centre of the osculating circle that is tangent to the ellipse and whose centre is the intersection of the principle diameters are determined by the straight line extensions of the sides of the triangles; the perimeter of the ellipse is made of circular segments that have their centres in the vertices of the rhombus.



Figures 1a and 1b: Images from the treatise of Sebastiano Serlio, I sette libri dell'Architettura

This construction draws on the first of the four methods proposed by *Sebastiano Serlio* (1475-1554) in his treatise *I Sette Libri dell'Architettura*, published between 1537 and 1551. In Book I Serlio proposes four methods for drawing an elliptical plan (Serlio 1545); the first of these calls for "two perfect triangles of equal sides joined together" (*si farà due triangoli perfetti di lato uguali congiunti insieme*). The vertices of the rhombus thus generated constitute the centres of the two pairs of arcs that form the oval. In his first method Serlio specifies that the rhombus be formed of two "perfect triangles", that is, equilateral, joined at the base, which serve to introduce the symbolism of the trilogy that had invested all parts of actual buildings but which had remained extraneous to the geometrical underpinnings of the church. The perfection of the geometric construction is further accentuated by Serlio's fourth method, which consists of drawing two circles that each pass through the centre of the other; the centres of the double pair of circles are represented by the vertices of the rhombus.
Finally, it is possible to inscribe in these last two circles two equilateral triangles with a common side: the shorter axis of the rhombus. One of the vertices of the two triangles is oriented towards the area of the presbytery, and the other towards the entrance of the church.
The geometric shape of elliptical domes is characterised by a hierarchical bi-axial symmetry, which significantly influences the stress state that is transmitted to the drum and the masonry below, because the transmission of stresses is not uniform. The horizontal stresses are variable, and if these are not countered with suitable abutments or other static devices, they can seriously compromise the structure, especially in correspondence to the major axis of the ellipse where the dome is usually characterised by an intrados whose curve is noticeably flattened.

HISTORY AND THEORETICAL TREATMENT

At the beginning of the seventeenth century, safety and stability of domes, regardless of shape, still relied on constructive traditions and on workers knowing the "tricks of the trade", with only fleeting mention made in the treatises of the time. In one of these treatises, *Vincenzo Scamozzi*'s (1548-1616) *Idea dell'Architettura Universale* published in 1615, reflections and suggestions of a theoretic and practical nature, gathered from the study of ancient Roman and Renaissance examples, are accompanied by innovative if elementary considerations about statics, thus presenting a view of morphological characteristics that acknowledges that the material used in construction constitutes a primary factor in the configuration of domes. Further, the treatise addresses the problem of the static behaviour of domes, arriving at innovative considerations such as those relative to the greater solidity of pointed vaults, and those regarding the use of significant wall thicknesses: because the greater the load bearing down on the vault from above, the more the sides are pressed outwards, Scamozzi notes that "from the feet to the sides the vaults can be made good and thick, and to tie them well back into the walls; but then from there up to the back of the vault you must be very careful, and make them very light and of high quality material" (*da piedi fino a' fianchi le volte si possono fare di buona grossezza, e unirle bene con le muraglie; ma d'indi in su verso la schiena della volta bisogna andare molto riservati, e farle assai leggere e di buonissima materia*) (Scamozzi 1615). This advice is even more interesting in light of the fact that it is chronologically close to the publication of the works of the French mathematicians of the second half of the seventeenth century, which soon defined with increasing precision the mechanism of failure and the role of friction in the statics of arches and vaults.
In fact, in the research in France between the seventeenth and eighteenth centuries, we can see that there is an increasing urgency to express ideas that had been up to then based on experience and practice on the building site in ways that were scientific and rigorous. Following the studies by *Philippe De La Hire* (1640-1718) and *Bernard Forest de Bélidor* (1698-1761), further progress in research about the structural mechanics of domes are found in the writings of *Claude Antoine Couplet* (1642-1722), who is able to state, in keeping with the theories of Leonardo da Vinci, that a semi-circular arch is in equilibrium if the force curve is contained within its thickness, equidistant between the intrados and extrados.
An analysis of the research conducted by French mathematicians and physicists shows that the approach to the study of the structural systems of domes changed radically over the course of the eighteenth century with theories by *Pierre Bouguer* (1698-1758) and *Charles Bossut* (1730-1814). Up to and throughout the seventeenth century, the dome had been in fact considered a unitary and compact structure, a kind of mysterious edifice that lifted itself as if by magic to cover other edifices, its complicated functioning unfathomable, about which it was possible to analyse only the scant information that could be measured. But the study of the laws that govern static behaviour implies the necessary decomposition of the structure as a whole into smaller elements; the dome is no longer an enigmatic unitary element, but becomes a structure subject to the application of mathematical and mechanical laws, treated like all the other parts of the building, since, as *Jean Baptiste Rondelet* (1734-1839) later stated, construction becomes an art when theoretical knowledge and practical knowledge preside equally over all its operations.
In this context there is a search for rules that are universally valid for both designing and for checking vaulted roofs, by means of investigations that take the shape of the diffusion of studies on the properties of the catenary curve, considered ideal for the stability of arches and vaults. Assimilated as the ideal line of forces and known to Renaissance architects, it is even used by Michelangelo.
Thanks to developments in the field of mathematical research, the passage from the seventeenth to the eighteenth centuries would see a multiplicity of studies on the theory of elastic sweeps. The results of the research of the Italians *Lorenzo Mascheroni* (1750–1800) and *Leonardo Salimbeni* (1752-1823), published between 1785 and 1787, are an indispensable premise to the formulation of the present-day theory for calculat-

ing domes. In the particular case of the elliptical domes analysed here, the truly original contribution is that of the architect *Antonio Bernardo Vittone* (1705–1770), who not only proposed new rules for their design, but applied them in some of his own projects (church of Grignasco).

ELLIPTICAL DOMES IN ARCHITECTURE

The concept of architectural structures derived from elliptical plans, with roofs constituted of systems of vaults or ellipsoidal domes, or by even more complex and imaginative systems composed of intersections of curved surfaces, makes its appearance only in the mid-sixteenth century, when architecture, having already resurrected and recreated all the daring feats of Roman architecture, looks for new expressive devices.

There are no examples of a similar problem in Classic or Roman antiquity, nor in Byzantine or medieval developments: there is the well-known hall of the Baths of Caracalla (212-217), whose plan comes close to an elliptical curve (being a perimeter generated by the conjunction of segments of arcs), but analogy with other well-known types indicates that it was conceived as covered by a groin vault flanked by apses with flattened curves. Roman architecture could not allow the asymmetry derived from this kind of architectural system nor, on the other hand, could construction practice founded on simple elements address the onset of the kind of construction problems implied in creating surfaces and connections that were geometrically complex, necessitated by transitions in vaulted roofs: bold, ingenious, plastic in plan and in elevation of centralised forms, in playing with the curves of exedras and vaults, they were in any case limited to schemes based exclusively on individual elementary geometric shapes or combinations of them.

In parallel with progress in scientific studies and the diffusion of treatises, elliptical domes began to characterise Baroque architecture, which in the full flowering of the Counter-Reformation, created buildings that were theatrical and evocative of the reaffirmation of the Catholic Church, in which, during the spectacular performance of ceremonies, the altars had to be clearly visible from every point in the space. The two fundamental types of plan are the longitudinal church plan, which derived from the traditional basilican layout – preferred by the clergy because it responded to the need for a space for the procession at the begin of Mass – and the smaller, central plan churches, – preferred by the architects in virtue of its use of the perfect shape – until arriving at the first attempts at a synthesis of the centralised and the longitudinal layouts. This objective was reached either by introducing two axes of symmetry into a longitudinal body, thus creating what might be called a *bi-axial space*, or by rendering the spatial elements mutually independent and, finally, by accentuating the continuity of the curved walls. This problem was solved by elliptical domes in longitudinal schemes, and excellent results were achieved as early as the sixteenth century: in Rome by *Jacopo Barozzi* da Vignola (1507–1573): Sant'Andrea in Via Flaminia (where this usually severe artist surprises us with his bold solution of placing the elliptical dome over a rectangle, connecting the four curves of the ellipse with pendentives) and Sant'Andrea dei Palafrenieri; by *Francesco Capriani da Volterra* (?-1588), in San Giacomo degli Incurabili; then with the works by *Gian Lorenzo Bernini* (1598–1680) and *Francesco Borromini* (1599–1667); by in Piedmont *Guarino Guarini* (1624–1683); and with Sanctuary of *Regina Montis Regalis* of Vicoforte, near Mondovì in Piedmont, built according to a design by *AscanioVitozzi* (1539–1615) with the exceptionally wide oval realized by *Francesco Gallo* (1672-1750).

Particularly representative of this new trend are the church of Sant'Andrea al Quirinale by Bernini – an transversal ellipse intersected by a longitudinal axis defined by an imposing entrance portal and an equally imposing presbytery – and the church of San Carlino alle Quattro Fontane by Borromini, which made space the element that forms the architecture, intended as a *unity* that can be articulated by not decomposed into independent elements; this unity in San Carlino is especially complex. The point of departure is the traditional longitudinal ellipse together with an broadened Greek cross plan: the two components are fused rather than combined, and give rise to an body that is bi-axially symmetrical. In this scheme Borromini's innovations find possibilities that are fully in keeping with his restlessness. His method of composition, based on a system of interwoven geometric figures and intersections of the surfaces of volumes, leads to an *optical deformation* that goes from pure geometric centrality to an organic elliptical shape. All of the members and surfaces of the internal skin undergo this process of elastic adaptation: the arches are bent into ruled surfaces, the coffers of the vaults follow the contractions and dilatations of the surfaces, the surfaces are inflected to reconstruct the continuity of the boundaries.

Borromini's ideas are taken up by Guarini, who used "open" spatial groups in his compositions: the (unbuilt) design for the church of S. Filippo in Casale, the Chapel of the Holy Shroud and the church of San Lorenzo in Turin, are the result of compositions made from geometric interpenetration in which first there are only polygonal shapes – prisms – and then rounded shapes – cylinders with circular or elliptical sections – that intersect with each other to determine vertices and movements of the walls that then perform in their turn analogous interweaving of spheroidal and ellipsoidal shapes.

Still in Piedmont, it was Francesco Gallo who was able to design for the drum of the Sanctuary of Vicoforte, built as we said by Vitozzi in 1596, one of the largest and complex elliptical domes ever built, with a diameter of 36 meters. The imposing buttresses placed by Gallo flanking the dome succeed in masking the asymmetry of the oval so that the effect comes close to that of a sphere, altering the soaring proportions conceived by Vitozzi.

A propagation of the Baroque in Piedmont by dint of shape and inspiration, as well as by the direct intervention of Italian artists, are the oval-plan churches of Vienna – which present many similarities with the Roman churches – as well as a group of buildings built in Bohemia between 1698 and 1710, which were evidently in-

spired by Guarini. More precisely, the relationship between the Bohemian and the Italian buildings is the result of Guarini's direct intervention with his design for the Theatine church of S. Maria Ettinga for Prague in 1769, and perhaps with his actually being in Prague.

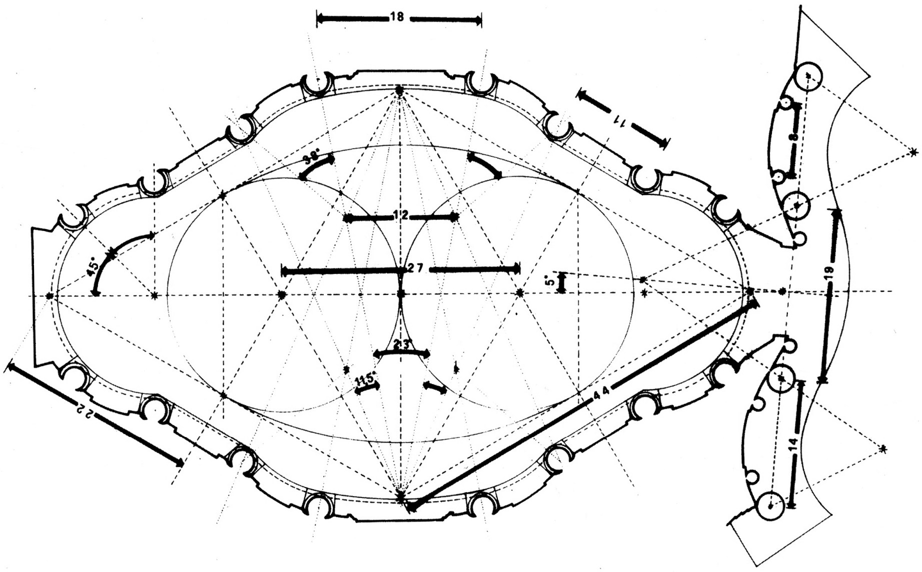


Figure 2: F. Borromini, geometric construction of the plan of the church of San Carlino alle Quattro Fontane in Roma

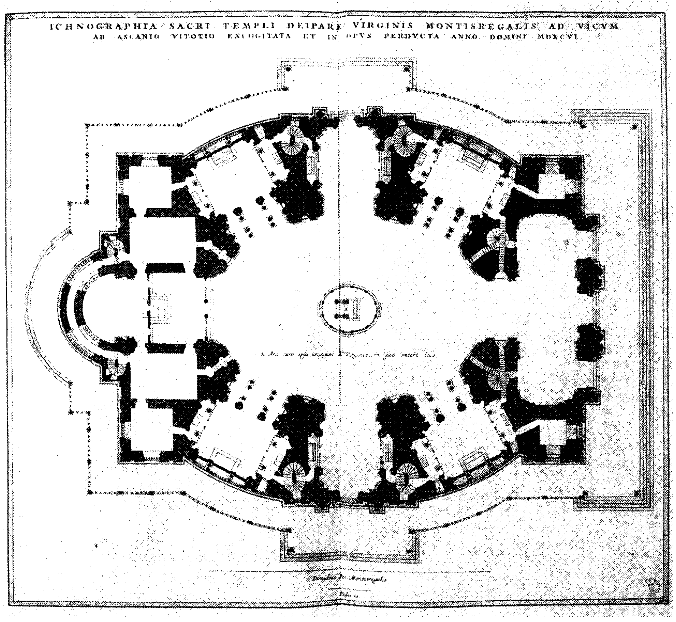


Figure 3: The plan of the Sanctuary of Vicoforte

In Genoa and its surrounding territory as well there are sacred buildings that have elliptical domes: the most significant structures were built by the Ricca brothers, the most well-known example being San Torpete, realized by *Gio.Antonio Ricca il Giovane* (1699-750) between 1730-1733. These, however, never achieved the monumentality of the buildings in the area of the Po valley. The reason for this lies neither in a lack of space in the cities of Liguria nor in the scant economic resources of those who undertook such projects. The difference lies in the diversity of the construction techniques used and in the availability of materials: in fact, in Liguria the wide-spread use of stone did not make it possible to build very large buildings, especially those with a geometry as complex as the buildings based on ellipses. These difficulties were more easily overcome in areas like Piedmont, where the predominant building material was brick, an unit that was regular and standardised, and which made it possible to build walls that were adequately compact, whose constitutive elements were sufficiently interlocked to be able to sustain stresses of structures that were more daring and less regular in their geometric form. In Liguria these problems were overcome by the mid-eighteenth century, with central-plan spaces based on an ellipse that were no longer capped by a single dome, but by pairs of transversal arches that connected a central vault to half-domes on either end.

ANALYSIS OF STATIC BEHAVIOUR OF THE ELLIPSE

The elliptical dome is such that the vertical reactions along the perimeter of the basic circular crown must turn out to be equal on the whole to the weight of the entire structure, and distributed along the base in a non-uniform manner, with the maximum values corresponding to the major diameter of the ellipse. Supposing for the sake of simplicity that the dome is divided into half-ellipses, there are two systems of forces that act on each of these: a) the resultant of the dead load of half of the dome applied at centre of gravity G; b) the resultant of the vertical actions relative to the half-ellipse and applied at its centre of gravity, which we can see does not coincide with the centre of gravity of mass "G". The two systems of forces are equal and opposite but are not aligned along the same line of action: in order for there to be equilibrium there must therefore be a moment between the two half-domes – exerted by the parallels, in compression in the upper regions and in tension in the lower regions – which is capable of balancing out the couple formed by the two resultants. This moment is not constant for all meridians, since the plan has different axes of symmetry in relation to the ellipse.
In order to give a brief picture of the complicated evolution of the problems and the solutions, we can look at the questions which the structural models examined up to that time do not answer and which thus require going beyond the model itself: what is it that is not yet clear in the methods set out by scientists of the eighteenth century, as elaborate and painstaking as they were? In the first place, the mechanics of collapse of the arch is not completely defined. Even given that the scheme of breaking into four elements is coherent with what is observed by experiment, it still remains to determine exactly where the breaks occur. In the second place, it is not clear exactly where the horizontal thrust is applied to the keystone of the arch.
We know that Coulomb *Charles Augustin de Coulomb* (1736-1806) dealt with the limit conditions of equilibrium without taking into consideration the material's compressive resistance; thus he was able to locate the horizontal thrust at the keystone at the extrados and the compressive force for the break at the intrados. *Claude Louis Navier* (1785–1836), on the other hand, wanted to refer the calculation to a more strictly defined limit situation, in which sections *Aa* and *Mm* are still effectively reacting with compressive forces that can be resisted by the material: it thus follows that the distribution of tensions can be at most triangular, with a null value in *A* and in *m* respectively (Fig.4a).
From this it results that the resultant of forces normal to the joint have to move from the edge that is most in compression by a distance that is equal to a third of the effective width of the joint, and that the force at this edge is double that which would occur in the hypothesis of a uniform distribution of the interior surface of the joint. These results, in which the influence of the tangential components and of the consequent deformations are completely abstracted, make it possible for Navier to calculate new values for the horizontal thrust at the keystone, greater than those that derive from strict or mathematical equilibrium relative to the hypothesis of an infinite compressive resistance, and which provide the means of ascertaining that the materials used in the design for an vault are not at risk for failure due to crushing.
Five years later appeared the work of *Franz Josef Gerstner* (1756-1832), which introduced for the first time two notions that were later widely used by scientists investigating arches: the *line of resistance* and the *line of force*. The first is a polygon that connects the centres of force on each of the planes of the joints; the polygon becomes a curve if the joints are infinitely numerous or slender. The second is the development of the lines of actions of the reactive forces from joint to joint. The two lines are usually distinct (Fig.4b). In order for there to be equilibrium, the *line of resistance* has to pass within the thickness of the arch; if it intersects the extrados at a certain angle, there is immediate breaking in the corresponding area; if instead it is tangent to one of the edges, rotation of the voussoirs is imminent and corresponds to the state of "restricted" equilibrium that can only be sustained by infinite material resistance. On the other hand, the angle with which the *line of force* intersects the joints has to be placed in relation to the angle of friction: if this is not close enough to a right angle then slide can occur. These ideas would be further developed and refined by *Henry Moseley* (1801-1872).

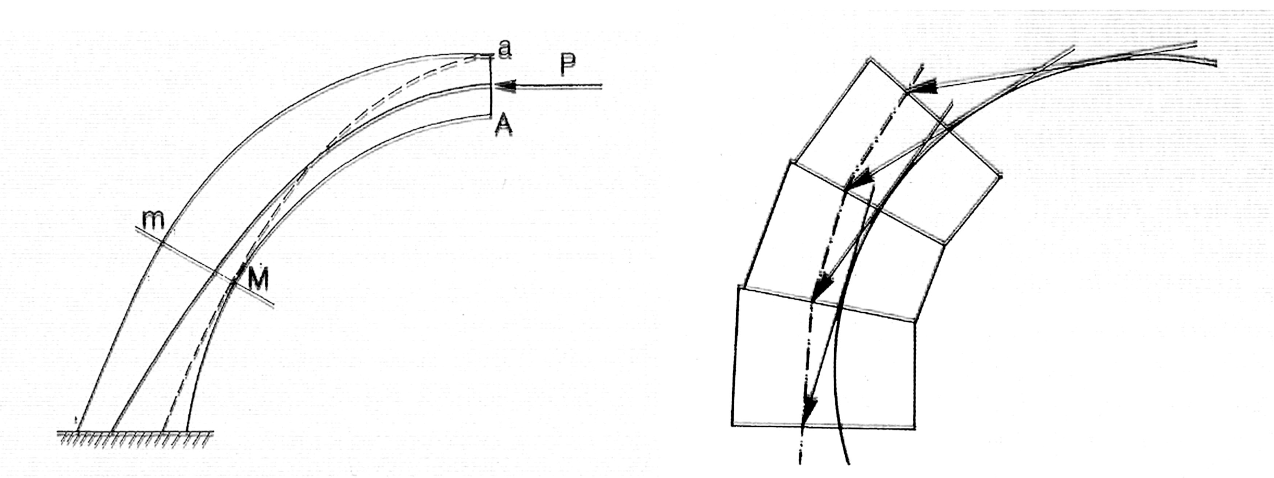


Figure 4a and 4b: Distribution of tensions by Navier and by Gerstner

In any case, Gerstner understood that, because of the hyperstatic nature of the problem, it is possible to draw infinite *lines of force* passing through various points of the keystone and tangents to different points above the impost that satisfy the equilibrium conditions. The problem lies in selecting those that are presumably closest to the limit configuration of equilibrium. However, Gerstner was not able to give the correct answer: he introduces ulterior hypotheses that turn out to be arbitrary. This failure is easily explained: without resorting to other considerations relative to deformation, there is no hope of adding anything to what is set by equilibrium. The prudent criterion of Coulomb, with its search for the maximum or minimum, is the only viable way, but its correspondence to reality remains uncertain.

In the case of the arch, H. Moseley was the main figure in the unsuccessful but fruitful attempt to add a new criterion of choice based on maxima and minima to the conditions for static equilibrium. As early as 1833 Moseley had introduced a *principle of least resistance* for the solution of hyperstatic problems; in 1839 he applied this principle to the statics of arches, observing that among all the lines of resistance that can be drawn starting from a generic point of section *Aa* of the keystone, the "true" one, which passes through the extrados in *a* and is tangent to the intrados at *M*, makes the value of horizontal thrust *P* a minimum.

Along the same lines as Moseley and in keeping with Navier's indications, it is then possible to attempt the use – in the spirit of the search for maxima and minima of a given function, in this case the force curve – a graphic method based on the determination of the funicular of the loads for determining the limit force curve, which must be completely contained in the thickness of the arch, better yet within its median third, so that none of its sections exhibit tensile stresses.

The value of thrust (*P*) in the keystone is determined in the hypothesis in no tensile forces are present in any section of the arch: this means that the line of resistance always remains within the arch's resisting section. Thus the force curve is not univocally determined, and to draw it we need only an elementary geometric construction, a funicular force polygon that has to respect certain conditions, such as, for example, that the thrust at the keystone must be horizontal and at most passes through either the extrados or the intrados.

The force polygon and the funicular polygon are thus the tools for determining our force curves, which defines the limit condition of equilibrium, that is, the condition before the formation of kinematics of collapse.

The section of the elliptical dome is considered as an elliptical arch constituted, in this particular case, to two circular arches, each with its own centre. Since the arch is symmetrical, and symmetrically loaded, for the purposes of graphic examination we use only a half-arch, since the other half behaves exactly symmetrically. Because this is a graphic procedure, we must make sure that the arch is drawn at a sufficiently large scale, and that is it divisible by a finite number of voussoirs. We then proceed to calculate the weight of the individual segments and apply that to the centre of gravity of each voussoir. The forces applied at the centre of gravity increase as we progress from the keystone to the impost; to identify the position and intensity of the resultant of the system of forces we use a funicular polygon, making use of the graphic construction described earlier. Once the position and intensity of the resultant has been identified, we proceed to look for the positions and intensities of the thrusts at the keystone and on the sections above the impost of the arch. Supposing that at the moment we know only the resultant R, both its intensity and its position, for the equilibrium of the half-arch it is necessary that the forces S, H, and R form a closed polygon and thus the forces have to converge in point O, which is identified as the intersection of the line of action of S with R; the connecting point where the force curve passes through the impost with O constitutes the line of action of thrust H.

Thus we proceed to determine the force curve, changing the position of the point of application of the thrust at the keystone and the position of the centre from which the lines of action of each force are projected as we go along.

Following Coulomb, we can proceed graphically by means of a computer algorithm that takes advantage of the computer's potential to reiterate this operation until the force curve identified is completely contained within the resisting section of the arch. In this way it is possible to determine the forces that represent the limit condition of equilibrium of the arch or the vault.

As we can see from the equilibrium polygon, the thrust in the keystone S, being composed of the resultant R of the load-bearing actions and of its own weight, gives rise to thrust H, and increases in intensity from the keystone to above the imposts; from this it derives that the arch has to have a thickness that increases gradually from the keystone to the impost.

Recalling that the arch is made of a material that by hypothesis is considered to be not resistant to tension, in order for the structural section to be completely in compression the force curve has to entirely contained in the central core of inertia of each of the infinite transversal sections of the arch. When this is not the case, it is necessary to increase the thickness of the arch, or to build it with thicknesses that vary. In the particular case in question, the force curve is tangent to the keystone at the intrados, and is tangent to the extrados in two sections that are slightly above the plane of impost, in correspondence to the ideal joint of the sides. When the load reaches the limit value, the mechanics of collapse are formed, which defines the kinematics of collapse of the elliptical dome. The dome "opens up" into wedges with a four-part collapse mechanism, in which the central portion tends to rotate outwards and the end portions tend to rotate inwards.

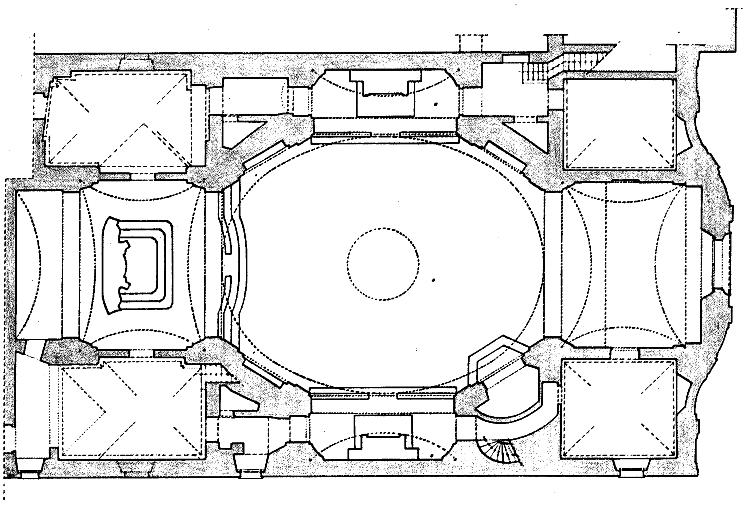


Figure 5: The plan of the church of San Giuseppe in Voghera (PV).

In one case studied – in the context of studies undertaken during a Master's thesis on the church of San Giuseppe in Voghera (Pavia), realized by Gio.Antonio Ricca il Giovane between 1740-1754, with architect Elena Tubino – the project architect avoided this problem, and picturing what would happen, solved the static "weakness" of the elliptical dome by stiffening it by means of ribs, and adding a substantial heavy reinforcement to its central portion to alleviate the phenomenon of outwards rotation.

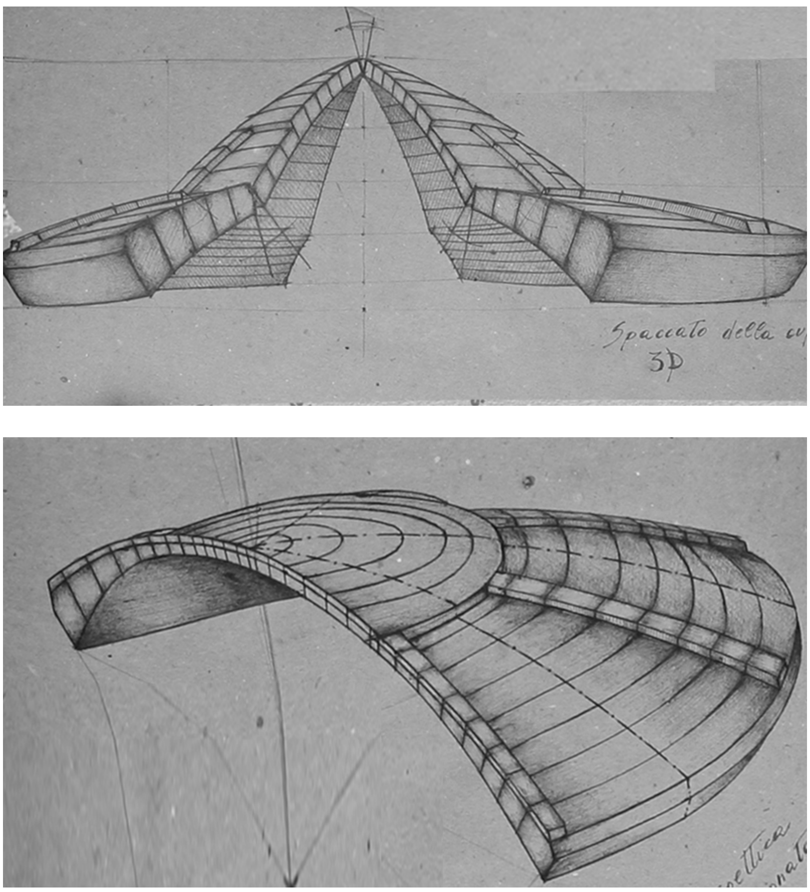


Figure 5a and 5b: The dome of San Giuseppe in Voghera (PV).

CONCLUSIONS

The study of the mechanics of elliptical domes has to take into account some particular aspects that must be evidenced and addressed with methodological and scientific rigour. The first aspect regards the relationship between architecture and structural mechanics, that is, the between form and structure, inasmuch as the par-

ticular typology of the elliptical dome – which is not a solid of rotation but rather belongs to surfaces with multiple curves even though still in the context of axially-symmetric geometries with hierarchical axes – recalls problems of the geometric layout and conformation of structural elements (voussoirs) that describe the constructive fabric of the structure. The surface itself, being a structure that transfers loads along a perimeter line, requires at the same time a buttressed structural system that fortunately lends itself to expression in Baroque architecture through the complex system of elliptical halls and side chapels.

To be sure, the complexity of the geometric system, even though facilitated by the various methods of layout given in the architectural treatises, makes explicit the equally complex construction technology and above all structural mechanics. From the point of view of mechanics, the study of static behaviour of elliptical domes has to take into account two distinct methodological approaches of which the one oriented towards the calculation of collapse has to address the complexity of the definition of the kinematics of collapse described by the intersection of the intersection of surfaces with different curvatures. Also, the complexity of the masonry fabric has to address possible hypothesis of the inability of the material to resist tensile forces, which cannot always be taken as a mandatory limit for directing the choice of the hypothesis of limit behaviour, but suggests that, alternatively, it is possible to predict situations tied to problems of mechanical resistance of the material rather than to final state limits.

In this sense, the example discussed in this brief paper regarding the church of San Giuseppe in Voghera, shows how the hypothesis of the kinematics of collapse of the structure, obtained by means of an analysis performed with an algorithm appropriately formulated to determine the limit kinematics, does not completely describe the situation of cracking confirmed during the examination performed as part of a restoration campaign, making it evident that in many situations related to events that are not referable to abstract and absolute theoretical hypotheses, it is possible to come across phenomena that cannot be described by an in-depth study of the resisting behaviour of the material. In this sense, this present paper suggests the necessity of various ways of approaching the problem of the mechanics of elliptical domes which contrast different methodological approaches relative to two distinct fields of calculus of collapse and of material strength.

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