The Evolution of Arches as Lightweight Structures - A History of Empiricism and Science

Neil K. Burford, Fraser W. Smith  
Dundee School of Architecture / Division of Civil Engineering, The University of Dundee, Scotland, UK

Christoph Gengnagel  
School of Architecture, University of the Arts, Berlin, Germany.

ABSTRACT: Arches have been an enduring structural form of traditional, Gothic and Renaissance architecture since the Romans brought them to prominence some 2000 years ago. However, it wasn’t until the Industrial Revolution and the advent of new, high strength, lightweight materials that a step-change was made to the form, span and construction of arches with the introduction of the first lightweight, filigree arch structure at Coalbrookdale in 1779. The advent of the tensile restrained arch occurred shortly thereafter due to the need to provide significantly larger span structures, more efficiently and economically. Over the next 100 years, spurred by the railway construction boom, there followed unprecedented innovation in materials and construction techniques. By the 1930’s, the advent of new steel technologies and simpler methods of analysis meant that the further development and applications for tensile restrained arches rapidly declined. More recently, there has been a gradual re-emergence of the tensile restrained arch in a number of significant architectural projects.

DEVELOPMENT OF THE ARCH

“ARCH (n). An architectural term: a material curve sustained by gravity as rapture by grief.” Ian Hamilton-Finlay, Bridge at Little Sparta, 2001

The arch is one of the oldest structural elements of traditional architecture. Historically it appears in two typical forms: the rigid masonry arch constructed from natural stones or bricks, or the flexible arch constructed from wood, bamboo and reed bunches (bales) (Figure 1). The masonry arch became quite important in the architecture of the antiquity and was first fully exploited by the Romans as the underlying structure for bridges, aqueducts and drainage systems. The form of the traditional arch developed empirically over a long period of time, the first common shapes being the circular arch, later replaced in the Norman period between (1066 – 1154) by the pointed arch, with the circular arch being re-used again during the Renaissance of antiquity. Throughout this time, aesthetics required ever increasing spans most evident in the great cathedrals of Europe such as at the 12th Century Cathédrale Notre-Dame de Chartres. A consequence of this was an increase in the slenderness of arches which resulted in more and more structures failing. The principal reasons were high thrusts at the abutments and horizontal settlement which could not be accommodated in masonry arches unable to carry bending and tension forces. The need for increased spans provided the impetus for Leonardo da Vinci (1452-1519) to undertake the first scientific study of the structural behaviour of arches. Hooke (1635-1703) and Gregory (1661-1708) developed the analogy between the catenary and the thrust line, which gave Poleni (1681-1761) in combination with Newton’s parallelogram of forces, the basis for his famous study of the cupola of the Saint Peter’s Dome at Florence circa (1419). Thereafter a simple graphical method to determine the thrust line for the dead load was developed. Further research by de la Hire (1640-1718), Coulomb (1736-1806), Navier (1785-1836), Bernoulli (1700 – 1782) and Poncelet (1788-1867) led to the underlying basis for contemporary structural engineering and finally, in 1869, to a method of analysis for three-pin, two-pin and fixed arches by Winkler (1835-1888).
DEFINITION OF A PLANE ARCH

An arch is a continuous-curved, planar, structural element normally lying in a vertical plane designed to support predominantly vertical loads (Plane Arch). A key feature of an arch is its ability to transfer external loads through mainly, compressive forces to its supports. Because this produces horizontal and vertical support reactions at both ends, the adequacy of the structure depends upon the ability of the supports to resist the horizontal components of force without excessive movement. If one end of the arch is not restrained horizontally the behaviour changes to that of a curved beam, carrying the loads dominantly in bending with the added problem that the span will change depending on the loading. This structure, which is statically determinate and simpler to analyse is often used in space enclosure applications in place of arches in situations where the foundations are incapable of resisting the horizontal loads. Plane arches are usually classified by their support conditions of which there are three principal types.

Fixed Arch

Fixed arches are the most efficient form of arch due to their stiffness, but rely on solid abutments to provide the necessary vertical, horizontal and rotational reactions. Since there are three reactions at each support making a total of six unknowns, only three equilibrium equations can be formed making the arch indeterminate to the third degree. A relative settlement or rotation of the supports can introduce additional significant stresses. The fixed arched system is typically used in insitu concrete arch structures due to the nature of the construction methods but is seldom used as solutions in lightweight applications especially mobile or transformable structures.

Two-Pin Arch

Two-pin arches are a common form of structure as the pinned supports allow the rotation of the arch at the ends under load, temperature changes and horizontal support settlements, making them relatively flexible and less prone to developing high bending stresses. With four reaction forces and three equilibrium equations these structures are indeterminate to the first degree.

Three-Pin Arch

Three-Pin arches contain an additional pin usually located at the crown of the structure. The additional pin allows the supports to move independently without developing bending stresses in the two halves of the arch. This structure is statically determinate. For this reason they are not sensitive to vertical and horizontal settlement or temperature changes. This feature and their relatively simple analysis were behind the widespread use of these systems from the end of the 19th Century until the Mid-20th Century. The use of a tie between the arch supports turns the structure of two pin and three pin arches into closed systems. This feature is useful where the supports are incapable of providing the required horizontal reactions, such as the situation of a station canopy supported on slender masonry walls. The arch in this case behaves as an independent unit, since the tie carries the horizontal component of thrust between the ends, leaving the supports to carry only vertical reactions. This type of arch is a self stable system that does not require horizontal support forces and is comparatively unaffected by vertical settlements of the supports. These forms of arch are common solutions for mobile and lightweight structures and can be used in conjunction with two-pin and three-pin arch configurations.
ENCLOSURE FORM

The geometry of an arch is often a compromise between structural and material efficiency and architectural considerations, such as a specific profile of enclosure, a determinate form of the building, traditional aesthetics or manufacturing and erection constraints. Commonly used shapes are (Figure 2.):
- parabolic arch
- circular segment arch (Roman)
- two-centred arch (Norman, Gothic, Arabic and transitional)
- three-centred arch (Basket Arch)
- transitionally curved arch (semi-ellipse, spline etc)

![Figure 2: Typical arch shapes defined by physical and ergonomic constraints](image)

BEHAVIOUR OF PLANE ARCHES

Although an arch carries its loading primarily in compression it may also be subject to bending depending on its form and loading (provided the material is capable of carrying bending). If an arch is made in the form of the mirror image of a hanging chain suspended under a certain set of loads, only compressive forces will develop in the arch in this case. For any other loading condition, the chain will adapt its shape to the new load. The thrust line of the forces in the arch will mirror the adapted shape of the chain when inverted and the eccentricity between the new thrust line and the centre line of the arch will produce bending moments in the rigid arch. While each load case produces a unique thrust line there are a number of dominant thrust lines that are often used for the design of arches.
- a parabola for uniform vertical load;
- a catenary for uniform load along the length of the arch;
- a circular arch for uniform radial force (such as hydrostatic pressure).

METHODS TO IMPROVE THE EFFICIENCY OF LIGHTWEIGHT ARCHES

With the absence of a dominant load case, such as the dead weight of heavy masonry arches, the design of arches for lightweight structures is determined by the intelligent handling of the bending caused by the deviation between the thrust lines and the main geometry of the arch. Slender arches are also susceptible to out-of-plane buckling but in the following this action is not considered and it is assumed that the arch is constrained against that action. A number of methods can be used either separately or in conjunction with one another to improve the performance of lightweight arches.

Shape Optimisation

Bending moments occurring under different load conditions can be reduced by optimising the curvature of the arch to suit the envelope of the thrust lines associated with the main load cases. However, this is not always possible in space enclosures where the enclosure section is more often determined by the ergonomics of the space, buildability and aesthetic considerations.

Cross Section Optimisation

For any element subject to bending, placing the material of the cross section away from the centre of the section (while providing adequate shear connection between the flanges) will make it more efficient. In an arch, this produces additional savings in materials and weight because the deviation between the thrust line and the centre of the section is less in an arch than in a beam for the same span.
Reduction of Support Thrusts

Arches are more efficient structures than beams due to their ability to carry load primarily as axial compression, but at the expense of requiring larger foundation abutments to provide the necessary horizontal reactions. The horizontal reaction at the supports is proportional to the total load and to the span, and inversely proportional to the rise of the arch. By making the arch as light as possible with a rise as high as economically feasible, it is possible to minimise the thrust for a given span.

Use of Tensile Restraint Systems

In a plane arch, large differences between the thrust lines and the main geometry of the arch will produce large bending moments. One method to significantly reduce the development of bending is to tie together or restrain points along the arch chord. This restraining system introduces a new set of forces that has the effect of reducing overall bending. At the same time, it effectively reduces the buckling length of the arch chord, and hence improves its efficiency under compression. With restraining systems, a greater degree of design freedom can be achieved, allowing the arch chord shape to be more easily manipulated for different ergonomic and aesthetic constraints. The mixture of an arch chord carrying principally axial compression and a restraining system carrying tension produces an efficient structural solution with reduced weight, increased stiffness and potentially lower cost. Such structures that minimise compression elements and maximise tension elements or tension fields, potentially provide the lowest weight, cost / volume enclosed (kg£/m3) with their essentially minimum and minimal use of materials. A large number of alternative solutions of tensile restrained arch enclosure systems are possible using this basic concept but with different structural, spatial and aesthetic criteria.

FROM ARCHES TO RESTRAINED SYSTEMS – AN HISTORICAL STUDY OF CONTEMPORARY TENSILE RESTRAINED ARCHES.

The first lightweight arches were designed by Philibert de l’Orme (1510-1570), constructed from a number of wooden elements (Figure 3). In contrast to the traditional ‘heavy’ masonry arch, his structures were made from short thick boards and achieved their stiffness using coupled circular and pointed arches in a single system. This basic construct was further developed by later designers providing the basis for a huge number of wooden trussed arch structures used in wide span structures at the end of the 18th and the beginning of the 19th century. During this period the design of arches would be revolutionised by the introduction of the new high strength materials: cast and wrought iron. The first arch bridge, constructed from high strength cast iron was built in 1779 in Coalbrookdale, illustrating the opportunities afforded by these new materials to build structures composed of discrete, thin, straight and curved, compression elements.

It was with the development of wrought iron with its higher tensile strength that made it possible for designers to develop the first truly lightweight filigree restrained structures, like the roof structure of the Theatre Français in Paris, (1786). This example clearly shows the attempts to control the stability of the new slender structure by tying intermediate points around the arch circumference using a multiplicity of tensile members. With better understanding of stability problems and the need for bigger spans to cover larger railway termini, arch structures became more and more lightweight and transparent. While the roof structure of the Station Gare de l’Est in Paris, (1847-52) comprises an addition of a comparatively simple tensile restraining system, the basic idea to support the arch at the quarter points shows a clear understanding of the stability problem and led to a very light structure. Following on from this project, some of most impressive examples of restrained roof structures were to arise with the arches of Victoria Station, London (1862), Lime Street Station, Liverpool (1892) or the Münchner Hauptbahnhof (1847-1849). In the latter two examples the efficiency of the restraining systems was improved by the introduction of tensile diagonals. The twin roof structures of the Schlesischer Bahnhof in Berlin, (1879-82) show the changes in arch design that appeared with the new method of analysis by Winkler for indeterminate systems and is one of the first examples of the application of rolled steel as a substitute for the more expensive wrought iron. Later, restrained arch systems were gradually replaced by two-pin and three-pin trussed arches. The opulence of form and the lightness afforded by the use of restraining systems was dis-
carded for the easily calculable, determinate and (arguably) economical solutions provided by trussed arches, although their supports had to provide the necessary horizontal reaction to maintain their stability. Independent from these developments, Vladimir G. Suchov (1853-1939) designed a number of highly efficient restrained arch structures. As well as designing some extremely innovative lightweight structures he invented a fundamentally new method to restrain arches by using diagonal tension members connected between the arch supports to points on the opposite side of the arch. Under asymmetrical loads the un-prestressed tensile elements restrained the outward deflection of the arch sides, thereby reducing the global deformation and the associated bending moments. This simple system not only provided a highly efficient means of restraining the arch using less material, but made the analysis easier by transforming the indeterminate two-pin arch into a determinate system for a load case where the diagonal tensile elements are slackening on one side only. The restrained arch developed for the glass roof of the exhibition hall at Novgorod (1895) was the first wide span structure designed in this manner and is a great example of creative engineering based on scientific analysis and knowledge. Freyssinet adopted Suchov’s system for the falsework of his great concrete airship hangars (1921-23) with a span of 88 m. Apart from the wooden roof structure for the railway station at Malmö, (1923), these elegant, economically re-strained arch structures simply stopped being developed for a long period before being reinvented at the end of the 20th century with the structures of Jörg Schlaich and Peter Rice et al. The main reasons for the decline of braced arch during this time may be due to two principal factors: the comparative simplicity by which two-pin, three-pin and fixed arches could be analysed meant engineers were reluctant to develop different and more complex systems, designing systems that were easily calculated; and the introduction of relatively inexpensive high strength standard rolled steel sections, allowed the realization of very large span arch structures which behaved as rigid truss beam systems, like the famous Galerie des Machines at Paris 1889. In 1989 Jörg Schlaich adopted the idea of a bicycle wheel for his restraining system for the grid shell of a glass roof at Hamburg. Following this, Peter Rice designed a series of braced arch systems, some of them based on the ideas of Suchov. The basis for the development of this latest generation of arch restraining systems lies in the intelligent use of prestress which increases the efficiency of the basic restraining system by reducing initial deflections and therefore the bending of the structural system resulting in very light and elegant constructions. Other than the creativity of engineers like Rice and Schlaich, the primary reason for this renaissance of new restraining systems has been the ability to more easily analyse complex indeterminate structural systems made possible by modern computers and the use of the new FE Analysis Methods. Additionally, a new focus on sustainable construction coupled with new directions in architectural aesthetics, forms and technology means that a greater emphasis has been placed on creative and experimental design and on structures employing lower material content and containing lower embodied energy.

**TYPES OF TENSILE RESTRAINED ARCH**

From the examples it is possible to derive a number of principle types of restrained arch (Figure 4).

**Hub Systems**

The restraining elements are disposed between points on the arch chord and one or more ‘fixed or free-hubs’ located within the area subscribed by the arch chord. Typically these solutions are used in roof enclosures where the arch is propped on walls or columns and the internal volume contained by the arch does not have a functional use e.g. atria and rail shed enclosures.

**Radial Systems**

The restraining elements are disposed between the arch supports and one or more points on the arch chord. These solutions are used in similar applications to the above but potentially provide higher levels of structural and material efficiency because the ties directly restrain the arch chord from a fixed support.

**Chord Systems**

The restraining elements are disposed between pairs of points along the arch chord. In these systems there is a fine balance between the degree of discretisation of the chord and the degree of trussing action provided by the ties. This form of restraining system tends to be quite efficient in terms of material use and ergonomics but less efficient structurally.

![Figure 4: Types of tensile restrained arch](image)
Truss Systems

These are the most widespread solution and the most diverse in terms of the arrangement of the elements. In all cases, the restraining elements closely follow the path of the arch chord providing improved ergonomics to that of radial and hub systems and better structural efficiency to that of chord systems. Another beneficial feature of truss systems is the flexibility in the position of the restraining system in relation to the arch, such that it may be positioned inside, outside or both sides of the chord depending on the ergonomic and structural constraints.

Mixed Systems

These systems combine one or more of the above systems in a single structure and are normally used in situations where there are a number of dominant load cases.

DESIGN TRENDS

A very large number of alternative solutions of tensile restrained arch enclosure systems are possible by applying the basic restraining system types using different structural, spatial and aesthetic criteria. Historically, restraining systems were used in many different forms depending on the loading, ergonomic, aesthetic, economic and buildability requirements of the enclosure (Figure 5). The first examples were developed during the period 1889 – 1923 to provide efficient, widespan enclosures principally for train sheds during the railway construction boom. The construction of so many different solutions was made possible due to the availability of new materials (cast and wrought iron) having higher tensile strengths than timber and the ability to form these into discrete, thin, linear and curved members that were also easier to connect together. The detail design of these early restrained arch systems adapted timber jointing technology and applied this to the element connection design without fully realising the potential of the new materials. However, known concerns with attaining consistent material properties (particularly tensile strength) and the unfamiliar nature of the materials meant that many of these early examples were over-structured with additional elements being used to help reduce material stresses. This is particularly evident in arch chord cross-section designs which rely on complex fabricated truss sections comprised of short members to reduce bending stresses in the chord section. Similarly the tensile restraining elements are comparatively short probably due in part to the difficulties inherent in fabricating long elements of consistent mechanical properties in wrought iron. As a result these early tensile restrained arches failed to fully capitalise on their true structural and aesthetic potential which in some cases produced inelegant and visually confusing solutions.

One of the principle differences between the early restrained arches and the later 20th Century examples has been in the development of high strength constant section steel profiles and high strength steel cables and their connections. These new materials and more sophisticated fabrication processes permitted the development of improved member section shapes to take account of the higher tensile strengths of the material. Modern, numerically based, cold forming and cambering processes now permit standard structural sections of various dimensions to be curved to single or multi-centred curves on the major (y-y) axis producing smooth accurate bends whilst maintaining the section geometry, essential for architectural steelwork. In parallel, modern welding technologies have significantly contributed to improved mechanical and aesthetic design allowing the fabrication of continuous chord sections without needing bulky and unsightly splice plates. These developments allow the arch chord to be easily and economically curved to a desired enclosure cross-section, while maintaining a high level of bending stiffness and consistent jointing between chord sections without the jointing becoming a dominant feature in the member profile. High strength and stiffness steel rope technologies currently used in the majority of contemporary restraining systems are made in a wide range of cross-section diameters and are low weight, visually light and provide consistent properties. These materials coupled with a range of standard end-fitting systems, make it easier and more economical to form connections between elements permitting much higher levels of prestressing. These reasons account in part for the recent rise in the large number of more complex truss type restraining systems. Truss systems combining thin arch chords carrying principally axial compression and complex triangulated restraining systems carrying principally tension produces efficient structural solutions with reduced weight, increased stiffness and potentially lower costs. They have the added advantage that they provide a greater degree of design freedom as the main geometry of the arch chord can be more easily manipulated for different ergonomic and aesthetic constraints while being able to control the stiffness of the structure through the use of an appropriate tensile restraining system that does not impede significantly on the internal space of the enclosure. With the very recent unprecedented development in new organic polymeric fibre technologies and other high strength materials, particularly composites and metal alloy technologies it is arguable that another step-change in the design of lightweight filigree tension restrained structures is possible. However, to date these materials have largely been used as substitutes for existing materials without fully capitalizing on their intrinsic properties or potentials.
Figure 5: Historical developments in lightweight, tensile restrained arches
REFERENCES


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