Evaluation of the Perpendicular Flat Vault Inventor’s Intuitions through Large Scale Instrumented Testing

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ABSTRACT: This communication describes an experimental setup devised to evaluate the structural behaviour of the flat vault in terms of thrust, ductility, loading capacity, failure mode and influence of design parameters. Three vaults are tested that differ by their span and the joint angle between the voussoirs. The instrumentation allows a continuous measure of the thrust in both directions and of the centre point displacement under load variation. The results are used to evaluate the relevance of the assumptions underlying the representations of the mechanical phenomena available to the inventors and allow validation and calibration of numerical models. These are then used to evaluate the effect of design parameters and to compare the efficiency of Abeille’s design with other more traditional flat vaults that implement converging joints in a one way or two way layout.

INTRODUCTION
The flat vault as invented by Joseph Abeille in 1699 (Fig 1a) (Gallon 1734), modified by M. Truchet the same year and developed and analysed by M. Frézier in his treatise of stereotomy issued in 1738 (Frézier 1738), can be seen as a special object testifying to the issues of architectural design and construction of the time, such as rational construction processes, relations between mechanics and stereotomy, or between aesthetics and mathematics.

Mainly in the perspective of contemporary reinterpretation of the principle, the flat vault has recently aroused the interest of a number of scholars in the field of construction history (De Nichilo 2003, Uva 2003, Fallacara 2006, Sakarovitch 2006), although the issues related to the key parameters of its structural efficiency have not to date received satisfactory conclusions.

Yet the mechanical phenomena involved are quite complex and cannot be correctly captured by the standard models implemented by today’s engineers and architects. In contrast to the intuition of modern designers, surely not enhanced by the extensive use of theoretical models, the constructive insight of the flat vault inventors could have anticipated these subtle phenomena to a certain extent.

The idea is here to evaluate the relevance of the assumptions underlying the representations of the mechanical phenomena available to the inventors. An experimental setup is devised to evaluate the structural behaviour of the flat vault and allows validation and calibration of mechanical models. These are then used to compare the efficiency of Abeille’s design with other more traditional flat vaults that implement converging joints in a one way or two way layout.

Of the invention’s presentation to the Academy of Science and of its approval, we know only what Gallon relates in his compendium describing the different machines approved by the Academy (Gallon 1734). The text speaks of the inventors as third persons, and allows itself criticism. There are three interesting points in the presentation of Abeille’s invention that are worth investigating.

First, the text specifies non argued proportions for the voussoirs that are in contradiction with the drawings (Fig. 1b). Second, the interest put forth for the invention which is presented with the continuous face on the ceiling side, is that the vault provides “all in one a ceiling for the lower storey, and a pavement for the upper storey. The rectangles of the extrados do not completely fill the upper surface, they leave voids giving way to a quite pleasant disposition, as since these voids draw small squares on the surface, it will be easy to fill them by small
pavements of the same size.” And third, the mechanical interpretation given in Gallon’s presentation is quite brief and not at all emphasized. The first part could apply equally to its timber equivalent (see below): “With this arrangement, each voussoir is carried on two others through its protruding cuts, and at the same time carries two others on its sloped cuts [...], this being reciprocal in all the vault’s area, it supports itself at level”. Nevertheless, Gallon (or the Academy, or the inventor himself ?) states that the vault “has the advantage that the thrust is shared by the four walls that support it, whereas for vaults in which the voussoirs are in standard cut, the thrust develops only on two sides”.

ELEMENTS OF CONTEXT

When trying to understand the mechanism of the complex assembly of identical voussoirs oriented 90° one to another, the image of Villard de Honnecourt’s interwoven timber arrangement naturally comes to mind (Fig. 2a). Variants of the concept are proposed by Leonardo Da Vinci in the Codex Atlanticus (Fig. 2b), and in 1545 Serlio shows still another scheme, in which each beam carries the ends of two others at mid span (Fig. 2c). In his treatise on mechanics (1669-1671), the English mathematician John Wallis analyses two kinds of such timber floors and computes the loads supported by each individual element.

The seventeenth century is that of important developments in stereotomy, making possible complex surfaces full of geometric shapes intersections and vaults of smaller and smaller curvatures. In science, the end of the century is marked by important contributions in the field of stability of vaults, and most significantly by the founding treaty of Philippe de La Hire in 1695, in which he proposes a method for the determination of forces acting within an arch, simply using the parallelogram rule and the assumption of frictionless joints. He entered the Academy of Architecture in 1687, and may well have had Joseph Abeille as a student.

The analysis of Abeille’s vault by Frezier forty years after its invention, even if based on very up to date scientific work in the field, still fails to extract complete coherence between Wallis’ computation for the timber floor and De la Hire’s treatment of the masonry arch (Fleury in press).

Figure 1: Abeille’s flat vault as presented in Gallion’s compendium (1734)
(a: figure from Gallon (1734); b: Proportions according to text)

Figure 2: Interwoven timber floor schemes by V. de Honnecourt (a), L. da Vinci (b), and Serlio (c)
EXPERIMENTAL INVESTIGATION

The experimental setup presented here is devised to evaluate the structural behaviour of the flat vault and to validate and calibrate numerical models for the investigation of design parameters. The experiment was conducted at the Grands Ateliers de l’Ile d’Abeau in France, and involved a number partners acknowledged at the end of the paper. Three vaults are tested that differ by their span and the joint angle between the voussoirs.

Geometry

The three specimens all cover a square area, and are composed of 49 voussoirs, according to Fig. 3. These are assembled together using a thin lime mortar joint including peripheric contact with the abutment system. The overall dimensions for the three specimens are 2.52m and 1.26m respectively for the large vault and the two small ones. While the proportions between thickness and total span are identical (1/14), the angle of the joint with horizontal is of 60° for the large vault, and respectively 75° and 45° for the small ones.

The big vault is assembled on a slightly curved centering, with a 3cm rise in the middle. Its smooth surface is placed on the extrados, unlike the small vaults which correspond to Abeille’s disposition.

Support

Vertical support is provided on the vault’s perimeter by profiled metallic sections, according to Fig. 4a. The horizontal abutment, independent from this first structure, is supplied by tubular metallic sections resting on pads so that the lower under face of the steel section is at a level with the intrados of the vault. These abutments are blocked horizontally by tie rods, two pairs in each direction, placed close to the edges in order to facilitate decentering (Fig. 4b). The pairs of tie rods are placed symmetrically with respect to the vaults thickness.

Instrumentation

The instrumentation is composed of a vertical displacement captor placed at the centre of the vault, and of steel plates equipped with strain gauges inserted in each tie rod, for the determination of thrust and relative horizontal displacements of the abutments.
Loading

The different loading sequences for the big vault are summarized in Fig. 5. The first stage consists of progressively lowering big sand bags, suspended to a bridge crane equipped with a dynamometer. This technique has the disadvantage that the loading surface slightly varies during loading. After unloading, the strain gauges are disconnected and the vault is lifted to higher supports in order to examine the intrados (Fig. 6). Reloading on the reconnected vault is then carried out first by stacking bricks, then by lowering the sand bags down to failure (Fig. 7).

For the small vaults, loading consists of a homogeneous brick stacking on the inner voussoirs, i.e. excluding the peripheral ring. The unloading of the small 45° vault is followed by the deposit of the tie rods, one direction at a time, so to test the capacity of the vault in pure bending.
RESULTS

Load versus thrust

The most complete and reliable results are issued from the testing of the large vault, for which the load vs thrust curve is given in Fig. 8. The maximum load supported is 6.5 tons (or 1166 kg/m²), including the 1.7 ton of the vault itself. This means that the vault can carry a live load of 2.8 times its own weight in this case. Taking modern building regulations, the corresponding service load would be 500 kg/m². It would therefore comply for any office, residential, hotel or hospital floor, including corridors, under the condition that the abutment system is sufficiently rigid. So as far as the load carrying capacity is concerned, Abellie’s invention works, at least for modest spans, since it is feasible and compatible even with our modern strength demand.

![Figure 8: Load versus thrust curve for the large vault](image)

The loading/unloading/reloading sequence shows a non-linear slightly hysteretic behaviour with a permanent increase in thrust at the end of unloading. The ratio of thrust over load varies according to table 1. This variation can be interpreted as the effect of different phenomena, such as initial downward effect of small initial prestressing, progressive damage of joint normal and tangential stiffness, variation of loading surface from brick to sand, initiation of damage in the stone. The implementation of the finite elements method for the modelling of the structural mechanics behaviour of the vault allows us to backup at least two of these assumptions.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Decentering</th>
<th>Sand load</th>
<th>Brick reloading</th>
<th>Final sand loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>0.12</td>
<td>0.67</td>
<td>0.46</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Damage process

The damage process is best captured in the test of the small vault with 45° joint angle, which was loaded almost to its limit (Fig. 9a). Cracks are observed that could lead to a collapse mechanism. Their patterns are in accordance with those that can be expected from a very short element subjected to non-uniform bending (Fig. 9b). Indeed, graphic statics methods applied to the vault show the possibility of the thrust line to pass through the joints but outside of the masonry, thus implying bending in the voussoir.

![Figure 9: Damage process for the small 45° vault](image)

(a: Cracking pattern on the small 45° vault; b: Loads, thrustline and plausible cracking pattern)
Numerical model calibration

In the numerical analysis, the model includes linear elastic voussoirs related to one another through joint elements and unilateral contact conditions. The value of the tangential stiffness of the joint is difficult to set beforehand. Its variation allows to evaluate the effect of that parameter as well as to calibrate the model in accordance to the experimental results. For a load evenly distributed over the entire surface and a very high tangential joint stiffness, the computed ratio of thrust over load is of 0.4. When the peripheral voussoirs are not loaded and the joint stiffness approaches zero, then this ratio climbs to 1.13. For predictive computations, one can set its value so that the limit angle of friction of 35° is nowhere attained. These results are in accordance with the experimental observations of table 1, and this allows us to have some confidence in the devised finite element model to study the impact of design parameters and to compare with more classic flat vaults configurations with converging joints, called ‘platbands’.

RELEVANCE OF THE INVENTION AND ITS DESIGN PARAMETERS

Proportions

An interesting thing about the invention’s presentation in Gallon’s compendium is that the proportions explicitly given in the text are in contradiction with the drawings (Fig. 1). This mismatch could be due to the same hesitation Frezier shows when he develops arguments about the best shape for the voussoirs: acute angles lower the thrust but weakens the voussoirs’ edges. Fig. 10 shows the variation of the thrust ratio for uniform loading when the angle of the joint varies. In accordance with the model of perfectly smooth joints prevailing at the time of Abeille’s invention, the numerical analysis shows that thrust does increase substantially when the joint approaches vertical.

At the same time, the indicator for tensile stress in the stone decreases, so does the vulnerability to cracking. In fact, compared to the proportions given in the text, those used in the figures lead to divide the self weight of the vault by 1.6 and the thrust by 1.4, while the tensile stress indicator is multiplied by 2.6.

It will be shown that if the design estimate of the thrust is half that of the platband, then the abutments are conservative even for the most vertical joints, a configuration which minimizes cracking vulnerability. The intuition leading to the hesitation is thus operative, even if the drawbacks of acute angles are more expressed in terms of edge fragility than tensile stresses in the stone due to shear and bending of the voussoirs.

Position of the pyramids

As we are used to see coffered ceilings and smooth floors, the fact that originally the vault is supposed to be built with the voided pyramids on the top arouses the curiosity of the modern builder. In the patent, the alternative is not discussed but the drawback is turned into the advantage of having an available pattern for elegant paving. The computations show that there is an important effect of the pyramids’ orientation on the thrust and its sensitivity to joint angle (Fig. 10). Considering the stone cutter’s idea that edge fragility is only due to acute angles, then the placing of the pyramids on top is relevant as it decreases the thrust. On the other hand, the thrust decrease is inevitably linked to higher shear and bending in the voussoirs, an issue that both Abeille and Frezier probably did not identify.

Figure 10: Effect of joint orientation on thrust ratio (for thickness, span and number of voussoirs according to large test specimen). (Results given by the same numerical model as calibrated according to test.)
Comparison with converging joints flat vaults (platbands)

Another point worth investigating is the idea that the thrust of the flat vault should be half of that of an equivalent one-way platband. The arguments put forth in the presentation of the invention and by Frezier are respectively: “there are four supports instead of two” and “the joints are oriented alternatively to the four sides [instead of two]”. These statements still hold for a two-way platband (Fig. 11). In terms strictly of thrust, the comparison with these two alternatives is shown in Table 2. (All the numbers given in that table come from numerical models, considering the same parameters, including those governing the joint behaviour. A simple hand computation for the one way platband gives a difference of 2% on the corresponding ratio.)

<table>
<thead>
<tr>
<th>Text proportions</th>
<th>Abeille 1 way platband</th>
<th>Abeille 2 way platband</th>
<th>Drawing proportions</th>
<th>Abeille 1 way platband</th>
<th>Abeille 2 way platband</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust (in N)</td>
<td>12 580</td>
<td>31 936</td>
<td>13 699</td>
<td>8 909</td>
<td>31 858</td>
</tr>
<tr>
<td>Weight (in N)</td>
<td>26 212</td>
<td>27 922</td>
<td>27 922</td>
<td>16 135</td>
<td>18 212</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.48</td>
<td>1.14</td>
<td>0.49</td>
<td>0.55</td>
<td>1.75</td>
</tr>
</tbody>
</table>

From this comparison, it can be concluded first that the old estimate for the thrust is conservative (if indeed the thrust for the one way platband is correctly evaluated) and second that Abeille’s design is relevant in terms of thrust reduction, and that was at the time the main concern. Furthermore, there are strong advantages for construction in the concept: only one geometry of voussoir; very basic stereotomy; decrease of self-weight; and minimization of rough stone volume.

CONCLUSIONS

The initial concept of the timber interwoven grid may have emerged from the lack of long beams, but calculations (not presented here) show that it is not relevant from the structural mechanics point of view. Its reinterpretation in stone clearly does not follow the same objective. It appears as an elegant brain teasing answer to functional, constructive, aesthetic and structural issues; reduced overall thickness, only one type of easy to cut voussoir, fancy pavement structure, and reduction of thrust.

On the structural mechanics point of view, Abeille’s invention turns out to be a relevant concept, at least for small spans, as Frezier had advised. The experiment has shown that in that case, its ultimate capacity is even compatible with the modern code specifications for heavily loaded building floors, as long as the abutment provided is stiff enough. This is an important result which may contribute to the renewed recognition of stone as a 3D structural material.

Whilst the arguments of Frezier for thrust reduction are specific to stone masonry, the important consequence in terms of bending and shear within the voussoirs are more specific to beams, and so escapes Frezier’s analysis (and so probably Abeille’s). The analogy with the timber ancestor and the lever interpretation that Frezier borrows to Wallis operates only as a conceptual machine to figure the laws of equilibrium. Nevertheless, the intuitions and models of structural mechanics of that time are seen to be operative in feasible and improved structural form generation, taking thrust as a criterion.
REFERENCES


ACKNOWLEDGEMENT

The experimental study described above was made possible thanks to a number of partners that must be acknowledged here. The Grands ateliers de l’Isle d’Abeau gave financial and technical support, and we are particularly indebted to Maurice Nicolas for logistical and photographic support. The work was carried out within the framework of a learning module with the financial support and students of the National Higher School of Architecture of Paris-Malaquay, and under the responsibility of Joël Sakarovitch. The teaching staff included Matthieu Pinon, Luc Tamborero, Giuseppe Fallacara, and François Fleury. Design and detailing of the experimental set-up is due to François Fleury, Joël Sakarovitch and Luc Tamborero. The entire stone cutting and handling skills were provided by Luc Tamborero, who deserves special thanks. On-site measures, for which the Laboratory of Form Analysis of the National Higher School of Architecture of Lyon gave financial support, were taken by the Civil Engineering and Environmental Engineering Laboratory of the National Institute of Applied Science of Lyon. Rough and pre-cut stone and voussoirs were gracefully provided by the Chevalier – 3D Pierre Company.