Baroque Timber Roofs without a Continuous Tiebeam

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ABSTRACT: In 2007, the authors have conducted a survey of baroque timber roof constructions in Southern Bavaria, and inspected around 40 church roofs (Holzer et al. 2008). An observation made was that a considerable percentage of the roof trusses analyzed are characterized by the lack of a continuous tie beam. We classify these roof structures according to their load-carrying characteristics and demonstrate, by some preliminary numerical analyses that different principal load-carrying mechanisms can be activated. Up to date only little research has been carried out to determine the actual load-carrying mechanisms of baroque roofs, which exhibit a complex three-dimensional behaviour. For a better understanding of the structural behaviour a full-scale loading test was performed on a typical baroque roof. Based on the results we suggest an improved non-linear model for traditional carpentry structures. In a last step and in view of a realistic computation, some data obtained by non-destructive test methods are presented.

CHARACTERISTICS OF BAROQUE ROOF TRUSSES

Whereas historical roofs of the Mediterranean region are usually of the “purlin roof” type, which exhibits a two-dimensional structural behaviour, the Central European roofs, by contrast, are typically based on a “rafter roof” principle. The group of the baroque roofs in Central Europe is – in contrast to the medieval roofs – characterized by a three-dimensional structural behaviour. This three-dimensionality depends on secondary structures which were originally introduced due to technical requirements of the assembly process, providing platforms for the assembly of large structures. These structures span in both transversal and longitudinal direction. Primarily, we encounter the so called “stehender Stuhl”, which was introduced around 1300, as well as the “liegender Stuhl” invented in the 15th century, which became the standard roofing solution of the baroque era. When the assembly process was finished, the “Stuhl” structures were not removed, but remained in the roof structure and became an integral part of it by combining its structural members with the basic rafter-tiebeam “trusses” to form “principal trusses” at regular intervals. For a better understanding of the “liegender Stuhl” we refer to Fig. 1, where the position of its elements and the definition of its technical terms are illustrated. It is obvious on first sight that the structural analysis of roof trusses with structures like a “liegender Stuhl” requires some attention to the longitudinal members, particularly since the “plate” and the “purlin” are usually connected to each other by wind braces. These braces in combination with the sloped struts provide a very stiff structure in the inclined plane, whereas the out-of-plane stiffness of this structure is negligibly low.

Another and the most significant feature which separates the baroque roofs from their medieval forerunners is – at least in Southern Germany – the lack of a continuous tiebeam (Holzer et al. 2009a). This circumstance is caused by barrel vaults (usually made of brick) which loom high into the roof truss. Probably more than half of all the major churches of the baroque era in southern Bavaria exhibit this situation. However, to guarantee the transmission of the thrust forces in the so called “open roof” (Ostendorf 1908, p. 90), a variety of solutions has been developed. The diversity of solutions ranges from simple “raised tiebeams” without any further precautions for carrying the thrust (Fig. 2, left), through several “reduced” variants of “scissor-braced” trusses, to fully “engineered” versions of the scissor braced truss (Fig. 2, right). Also, a number of hybrid forms, combining elements of the three basic types mentioned, were developed.
Stressed by the effects of age, many baroque roof structures require fundamental and comprehensive rehabilitation. On the other hand, such work should be limited to a minimum of loss of the original structure by recognizing baroque timber roofs as a technical heritage (ICOMOS, 1964; ICOMOS 1999). The current state of structural analysis and the current design codes do not reflect this goal adequately. So we want to focus our attention on a more detailed analysis of baroque timber roofs.
A QUALITATIVE CASE STUDY OF THE LOAD-CARRYING MECHANISMS

As an example for our analysis the roof of the former Augustinian abbey church at Weyarn, some 20 km south-east of Munich has been selected. We have chosen this roof because of it is well representative for a larger class of baroque roofs. The structures of the “stehender Stuhl” and “liegender Stuhl” are combined, it has no tie-beams at the base level of the roof, and last but not least a “raised tiebeam” with a reduced “scissor braced” system is integrated. Furthermore, it has not been marred much by modern restoration works, and the roof is very systematic in its general structure (Fig. 3).

In a first step, a simplified linear elastic analysis of the principal transversal frames of the roof is run. Therefore, the actual loading conditions will be neglected, and we treat the three-dimensional behaviour exclusively by introducing spring supports for the longitudinal roof members (Blass et al. 1997). It should be clearly mentioned that this preliminary analysis does not reproduce the actual stress state of the structure. The key features of the roof of Weyarn are the following: First, the “stehender Stuhl” frame in the lower storey provides only an elastic support for the transversal frames, particularly because the transversal frames are not aligned with the axes of the buttress-walls. A simple linear elastic computation of the "stehender Stuhl" as a perfectly hinged truss yields a spring stiffness of approximately 5 MN/m for these supports. For our parametric study the limiting values of minimum 2 and maximum 8 MN/m are applied. Secondly, the structural behaviour of the roof also depends on the connection between the two “diagonal braces” and the king post. Therefore, we bracket the influence of this detail by two extreme assumptions. Once, we consider the braces to be perfectly fixed, and once we remove the joints completely.

The results of our study are presented in Fig. 4. The loadings are proportional to the dead load. In Fig. 4a, we portray the results obtained from our system with a spring stiffness of 2 MN/m. Under these conditions, the transversal frame performs like a perfect truss with a divided, angled lower chord (formed by the two diagonal braces), a broken upper chord (the rafters), and a vertical tie (the king post). All the other members of the roof receive only very little stress, including the “raised tiebeam”. By contrast, when we choose the maximum value of 8 MN/m for the spring constant, we get the results depicted in Fig. 4b. In this case, the diagonal braces and the lower collar beam receive tensile stress, and the principal load carrying mechanism changes to a polygonal arch constituted by the struts of the “stehender Stuhl” and the “liegender Stuhl”. The arch also establishes itself if we select some average value for the spring stiffness of the longitudinal frames, but remove the joints between the diagonal braces and the king post.

The “true” structural behaviour under dead load must be a mixture of these two load-carrying systems. The redundancy indicates a higher level of structural safety. For a usage in rehabilitation practice, this qualitative assessment of the structural safety must be complemented by a quantitative one. Certainly, the selection of the points of interest for the parametric study was still quite random. We could as well have played with the stiffness of the joints between the rafters and the king post at the apex of the roof or with the horizontal stiffnesses of the supports. And keep in mind that other types of loading than symmetrical dead load may cause different load-carrying reactions. All these considerations led to the decision to perform a full-scale loading test at Weyarn.
A FULL-SCALE LOADING TEST AND ITS INTERPRETATION

In order to determine the true behaviour of a typical baroque structure, the roof at Weyarn has been subjected to an in-situ loading test (Holzer et al. 2009b). For this test, the principal frame located exactly in the centre of the roof, as well as a common rafter located next to it have been selected. The loads in form of students of civil engineering from our university were applied on the collar beams as “point loads” with a magnitude of around 1 kN. The positions of the single loads are pictured in Fig. 5, the precise setup of all load cases is listed in Tab. 1. On the principal frames, a total load of around 12 kN was achieved, whereas the collar beams of the common rafters were only loaded by a total of 10 kN. The deflections were measured at 4 common rafters by a programmable high-precision laser-based tachymeter at 62 measuring targets which had been installed previously (Fig. 6). At regular intervals between the loading cases, the unloaded structure was measured in order to catch possible residual deformations or external influences e.g. caused by temperature. It turned out that we could not observe any permanent set, but all deflections were reversible. It also turned out that the deflections were quite limited on the entire perimeter of the “arch” structure constituted by the “stehender Stuhl” and “liegender Stuhl” frames. In all cases, the maximum deflections occurred on the lower collar beam, which was 13.8 mm in the centre of the beam of the principal transversal (point X in Fig. 6) for the symmetric load case 1.

A very interesting observation made was that the sum of the displacements associated with the two non-symmetric load cases did not reach that value of 13.8 mm. It was significantly lower, just 9.6 mm. This means that the system behaved less stiff for the symmetric loading. A repetition of the symmetric loading case provided the same result. On the other hand, we could not find such an effect on the collar beam of the common rafter. The sum of the displacements in point Y, Fig. 6, for the two non-symmetric load cases was almost exactly the same as the displacement under the full symmetric load. Upon closer inspection of the structure, we detected gaps in nearly each joint. For instance none of the tenon- and mortise connections are fitting perfectly (Fig. 7, left); they have gaps at their end and either at the top or bottom. These gaps can be attributed to assembly constraints, some other gaps are probably due to wood shrinkage, particularly in the lap joints or others may even be attributed to past overloading events causing permanent deflection. The gaps may play a key role in the non-linear structural behaviour. However, from the reversibility of the deflections caused, it is evident that possible friction or slip effects in the carpentry joints cannot be a major influencing factor.

The main difference between the principal truss and the common rafters is the presence of the diagonal struts. These diagonals are “lap-notched” across the collar beam (Fig. 7, right) and in case of a bending deformation, the crossing with the diagonals acts as an elastic rotatory spring. While in the symmetric loading case, the rotation of about 2° required to activate the springs has not been reached, it has been achieved in the non-symmetric loading cases. Based on this knowledge, the analysis of the loading test can be boiled down to the influence of only one single isolated joint, and it is immediately obvious that the observed general “closing gap” behaviour of all traditional carpentry joints has a marked effect on the entire load-displacement behaviour of the structure and leads to load-dependent changes of the system.
Table 1: Load cases of the full-scale loading test.

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Load application points</th>
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<tbody>
<tr>
<td>1 and 7</td>
<td>A, B, C, D, E, F, G, H, I, K, M, N</td>
</tr>
<tr>
<td>2 and 8</td>
<td>a, b, c, d, e, f, g, h, m, n</td>
</tr>
<tr>
<td>3</td>
<td>D, E, F, K, M, N</td>
</tr>
<tr>
<td>4</td>
<td>d, e, f, m, n</td>
</tr>
<tr>
<td>5</td>
<td>A, B, C, G, H, I</td>
</tr>
<tr>
<td>6</td>
<td>a, b, c, g, h</td>
</tr>
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For demonstrating an adequate modelling concept of this non-linear behaviour of the joints we want to take a look at a simple example, an orthogonal crossing between two timber members (Fig. 8, left). In a first step the beams will rotate with respect to each other until their lateral surfaces come into contact. The rotation is only impeded by the torsional stiffness of the wooden pin (usually oak), whose average diameter is usually about 2 mm bigger than the circular borehole. At a second stage, once the parts come into compressive contact, the stiffness of the joint increases enormously. Fig. 8, right shows the characteristic load-deflection relation for a typical lap joint based on the supposition that we have a linear distribution of the compressive stress along the contact surfaces. Even though the stiffness of the nail is almost negligible, it suffices for reducing the deformation to zero again once the load is removed. Practically all the joints in the structure will have similar load-deflection behaviour, since gaps are ubiquitous. Until now all research projects on the structural analysis of historical roofs (such as Deinhard 1963; Vogeley 1981; Görlicher 1999; Kraft, 1998) did not consider these non-linear effects and therefore came naturally to the result that baroque constructions had in fact a rather low safety level. With a view to an appreciably increasing interest in obtaining more realistic factors of safety, our model describes for the first time a structure which changes with changing load levels, which is in turn a realistic, non-linear behaviour pattern of traditional carpentry.
MATERIAL DATA OBTAINED BY NON-DESTRUCTIVE TESTING METHODS

With regard to a realistic modelling and analysis we also need to obtain real-life input data for the materials encountered. In order to affect the historical structures as little as possible, we decided to adopt non-destructive testing methods in the roof of the church of Weyarn. First and foremost our interest was to establish a working value for the modulus of elasticity $E_0$ in the fibre-parallel direction, which is one of the most important parameters for further numeral analyses. The prevailing investigation method is based on the measurement of the velocity of a transmitted low frequency ultrasonic wave (22 kHz) in the longitudinal axis. The so called dynamic modulus of elasticity follows by

$$E_{dyw} = v^2 \cdot \rho$$  \hspace{1cm} (1)$$

and is about 7 percent higher than the required modulus of elasticity $E_0$ (Tichelmann et al. 1993). Steiger 1996 analyzed the scope and limits of this method by testing Swiss spruce wood on its qualities for civil engineering.

In our field study, 212 single measurements of the velocity have been taken with the apparatus Sylvatest Duo, which resulted in a mean value of 5559 m/s and a standard deviation of 342 m/s. Our data passed a $\chi^2$ test for fitting a normal distribution. However, in order to solve equation (1), we also need to know the density of the inspected timber construction. The non-destructive testing method used for determining that value relies in turn on the measurement of the penetration $t$ of a steel bolt with a diameter of 2.5 mm. The apparatus employed was Pilodyn 6J, for which an identification procedure was provided by Görlacher, 1987. After a first arithmetic averaging process of every four of a total of 240 values taken, the remaining 60 values were also tested for their distribution. Another $\chi^2$ test of goodness of fit rendered the expected Gaussian distribution with an expectation value of 14.3 mm and a root mean square deviation of 2.11 mm. In Fig. 9 the expected and observed frequencies of the different penetration classes are illustrated. The density is then given by the following linear transformation of the measured penetration:

$$\rho = 795 – 27,1 \cdot t$$  \hspace{1cm} (2)$$
Now, by using the calculation rules for expectancy values and for variances, we find the expected value for our density of $408 \text{ kg/mm}^3$. The root mean square deviation is $57.1 \text{ kg/mm}^2$.

In a last step, inserting our velocity and density results into equation (1) and considering the 7 percent decrease mentioned above, we are able to establish a modulus of elasticity $E_o = 11770 \text{ N/mm}^2$ with a root mean square deviation of $1426 \text{ N/mm}^2$. By means of these parameters, we can now assess all kinds of quantiles or fractiles, and, based on the 5%-fractiles, we are able to classify our timber structure in accordance to the strength categories of the valid design codes.

![Figure 9: Expected frequencies, E (class) and observed frequencies, H (class) of the different penetration classes](image)

**CONCLUSIONS**

After exploring the key features of baroque churches in Southern Bavaria and a classification based on that knowledge, we have demonstrated significant three-dimensional load-carrying effects and discovered a pronounced sensitivity to changing load cases by a simple case study. For a better understanding of the load-carrying mechanisms an in-situ load-carrying test was arranged at the church in Weyarn. As a final result a characteristic non-linear behaviour of the joints affecting the whole structure was found. In a last step, some data for the modelling process were collected and presented.

Currently, the authors are working on a practical implementation of the rotational, tensile and comprehensive models into a finite element code developed by the second author, which is based on Timoshenko beam theory and a $p$-type (higher order polynomial) approximation. Timoshenko theory is particularly appropriate for timber constructions because the shear stiffness of wood is low in comparison with the modulus of elasticity due to the anisotropic behaviour of wood. Inspired by the actual construction process of the traditional carpentry structures, which is based on an assembly of different parts which are essentially plane, we want to restrict our attention to the plane sub-structures and otherwise neglect all out-of-plane effects; then, we can rotate all parts in one common plane and connect them to each other by simple geometric compatibility constraints. Such a model requires only the availability of a "coupling element" which has no spatial extension, but can be used to couple the displacement components of arbitrary points of two structures to each other. A straightforward implementation in any plane frame finite element analysis tool can be done without major difficulties. Somewhat more complicated coupling elements are also required in order to introduce the "gap based model" for the connections. The necessity of such spring-elements for modelling joints even in the linear case is already mentioned in Hauer, 1993. In a next step, we want to record some average or typical gap widths for the different connection types encountered in a particular roof with the aim to create an universally valid and realistic model for the stiffnesses of the carpentry joints. On the basis of such an extended non-linear model, a more realistic assessment of the actual load-bearing behaviour and safety of traditional carpentry structures will hopefully be possible.

**REFERENCES**


