The Evolution of Structural Design of Monumental Vaulting in Opus Caementicium in Imperial Rome

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ABSTRACT: We present a numerical study of the structural behavior of monumental Roman cross-vaulted halls in opus caementicium under static gravitational loads. The study is based on linear elastic FEM stress analysis and is focused on the Great Hall of Trajan’s Markets and the Frigidarium of the Baths of Diocletian. Both cross vaults were designed following a similar supporting scheme based on contrasting arches, transverse shear walls, and supporting blocks. There are, however, critical differences in the two structures, which allow us to evaluate the shift in design paradigms that took place after the construction of the Great Hall. The analysis of the Great Hall reveals the inherent weakness of the support system. The contrasting arches play no significant role in the static equilibrium of the vault. The shear walls and, in particular, the supporting blocks are the critical elements on which the stability of the vault hinges. In fact, motion of the blocks might have caused a near collapse of the vault. The analysis of the Frigidarium shows a much improved structural configuration. The shear wall is extended upward, the contrasting arch is lowered and becomes an integral part of the shear wall, and, most importantly, the supporting blocks are now completely encased in the opus caementicium. This suggests that Roman engineers were able to detect and correctly interpret the structural deficiencies of the Great Hall, thus developing the knowledge necessary to build the gigantic hall of the Frigidarium.

INTRODUCTION AND OBJECTIVES

In little more than two hundred years – beginning under the emperor Trajan and terminating with Constantine – Imperial Roman architecture produced a series of monumental cross-vaulted halls built entirely with pozzolanic concrete (opus caementicium). Two outstanding examples are the Great Hall of Trajan’s Markets and the Frigidarium of the Baths of Diocletian, both in Rome and in excellent state of conservation (Fig. 1). Built between A.D. 98 and 117, the Great Hall is the earliest known free-standing cross-vaulted hall in opus caementicium. The hall is a rectangular space approximately 36 m. long by 9 m. wide, covered by a system of six cross vaults forming a continuous concrete block. The vault is carried on travertine corbels by lateral shear walls and is connected to the adjacent structures by lateral contrasting arches. The Frigidarium of the Baths of Diocletian (A.D. 298-306) is one the latest gigantic halls built in Rome near the end of the empire. The hall, about 63 m. long and 20 m. wide, is partitioned into three bays, each covered by concrete cross vaults, carried by monolithic columns and laterally stabilized by contrasting arches resting on shear walls. Because of their chronology and the excellent state of conservation, the Great Hall and Diocletian’s Frigidarium are of extraordinary importance for understanding how Roman engineers developed the design paradigms for monumental concrete vaults. Although these monuments have been studied extensively by archaeologists and architectural historians, the engineering analysis of the vaults and their structural support system has been limited to the early study by Giovannoni (1929) and the recent investigation on the seismic response by Croci et al. (2008), both addressing the Great Hall only.

As part of an interdisciplinary research on the engineering design of concrete Roman vaults conducted in collaboration with the Museums of the Imperial Forums and the University “La Sapienza” in Rome, we are investigating the structural behavior of the Great Hall and of Diocletian’s Frigidarium through stress analysis based on the Finite Element Method (FEM). In the present paper, we limit the discussion to the response of the structure under static gravitational loads and under the assumption of linear elastic behavior. The primary objectives are: (1) develop an understanding of the mechanics of deformation of the vault and its supports, (2) evaluate the structural design of the two systems, and (3) identify the connections between the two structural solutions. This work sets the basis for future studies aimed at understanding the mechanics of failure of concrete cross vaults in the post-critical domain.

Figure 1: The Frigidarium and the Great Hall shown at same scale

PREVIOUS STUDIES

The Great Hall has been studied extensively and its importance in the history of architecture recognized since being brought back to its original state by the 1926-1934 restorations (MacDonald 1965). (Ward-Perkins 1981). Giovannoni identifies the lateral arches as examples of flying buttresses, functionally identical to those found in medieval gothic architecture (Giovannoni 1913, p. 287; 1925, p. 63). This identification is made in the context of a study of Roman prototypes of contrasting arches and flying buttresses, in which Giovannoni draws a direct analogy between the structural system of transverse walls and contrasting arches found in the Frigidarium of the Baths of Diocletian and the Basilica of Maxentius, and the system of transverse walls and flying buttresses of the Great Hall. In a later study Giovannoni applies the static graphic approach on a transverse section of the hall to show that the vault thrust line does not follow the intrados of the vault, but, due to presence of the lateral arch, crosses internally to the projecting corbel and then remains inside the transverse wall (Giovannoni 1929, p. 237; 1938, p. 307). Although based on incorrect dimensions, this analysis is historically important since it remains to the present the only one ever published on the presumed functionality of the arches.

A different structural interpretation was put forth by MacDonald, who correctly relates the functionality of the contrasting arches to their position within the geometry of the vault and to the mechanical behavior of concrete (MacDonald 1965, pp 87-88). In discussing the lateral arches he rejects Giovannoni’s interpretation and asserts that “...their position and the structural nature of the building preclude calling them flying buttresses. The near-monolithic quality of the vault is such that once the concrete had cured the arches would be brought into play very little and probably not at all.”

In a series of extensive studies dedicated to concrete vaulted construction, Lancaster relates the necessity of horizontal buttressing for the Great Hall to the use of traveertine blocks to support the vault (Lancaster 1996; 2000; 2005). In a return to Giovannoni’s interpretation, the arches are described as early precursors of flying buttresses designed to convey the lateral thrusts produced by the cross-vaults into the extension of the transverse walls (Lancaster 2005, p. 136). In a recent comprehensive treatment of building engineering and construction, Addis continues to interpret the arches as flying buttresses but provides only a qualitative sketch showing vault thrusts flowing through the arches into the shear walls (Addis 2007, pp. 49-50). Croci and co-workers address through FEM analysis the seismic behavior of the vault and its supports before and after the extensive reinforcement introduced during the 2005-2007 consolidation of the monument (Croci et al. 2008).

The Baths of Diocletian, one of the finest surviving examples of Roman architecture and structural engineering, awaits a modern comprehensive study. The old and detailed reconstruction by Paulin (Casanelli et al. 2002, pp.172 -179) still provides useful information. As indicated above, Giovannoni draws a direct analogy between the structural systems of the Frigidarium, the Basilica of Maxentius, and the Great Hall (Giovannoni 1913; 1925).

In discussing the present church of Santa Maria degli Angeli e dei Martiri, which includes much of the Frigidarium, Micozzi provides valuable plans and elevations (Micozzi 2005) and De Falco gives important details on the status of the Roman vaults (De Falco 2005). Lancaster provides comprehensive information on the material used in the construction of the baths, and, in particular, on the various types of aggregate present in the opus caementicium (Lancaster 2005). Although focused on different monuments, DeLaine’s study of the Baths of Caracalla (DeLaine 1997) and Samuelli Ferretti’s analysis of the Basilica of Maxentius (Samuelli Ferretti 2005) contain much material on the opus caementicium, the structural behavior, and the FEM modeling of great value for the present study. We are not aware of any engineering analysis of the vault of the Frigidarium.
METHODS

The stress analysis is performed using three-dimensional linear FEM models representing either the entire structure or modular sections subjected to static gravitational loads. The opus caementicium is assumed to be fully hardened and behaving as a linear elastic material. In an extensive experimental study Samuelli Ferretti and coworkers have shown that opus caementicium has a mechanical behavior qualitatively similar to modern concrete, characterized in compression by elastic deformations approximately linear up 70% of the strength limit, followed by a pseudo plastic region (Samuelli Ferretti 1995a, 1995b, 1997); (Perno 1997). A similar behavior is shown in tension, but in this case the strength is approximately one tenth of the compressive limit. Following (Samuelli Ferretti 2005), we model opus caementicium as a linear elastic material with Young’s modulus $E = 3$ GPa, mass density $= 1500$ kg/m$^3$, and Poisson’s ratio $= 0.2$. The compressive strength is assumed to be between 4 and 5 MPa and the tensile strength between 0.4 and 0.5 MPa. FEM models have been used for the stress analysis of un-reinforced concrete structures, such as dams, since the early development of the FEM, see, for example, (Zienkiewicz; Taylor 2005). An exhaustive discussion on the applicability of the method to opus caementicium is provided in (Samuelli Ferretti 2005). Examples of systematic FEM analysis of major ancient Roman monuments built with opus caementicium can be found in (Mark; Hutchinson 1986), (Tosi 1997), (Croci 1998; 2008), and (Samuelli Ferretti 2005).

STRUCTURAL ANALYSIS OF THE GREAT HALL

Geometric Modeling and FEM Meshing

The transverse section and the structural solid model of the Great hall are shown in Fig. 2. The dimensions are taken from a survey of Trajan’s Markets by La Sovraintendenza BB. CC. del Comune di Roma. The vault itself is built as a single concrete block, flat at the extrados and with the intrados articulated into a series of six approximately equal cross vaults. The vault is supported vertically by fourteen travertine piers, each consisting of two superimposed elements: a projecting corbel above a prismatic block partially embedded into a concrete wall. These walls are set transversely to the long axis of the hall, following a nearly symmetric pattern. Above each pier and on the plane of the corresponding transverse wall, a lateral concrete arch horizontally connects the top of the vault to the vertical extension of the wall.
The structural model contains several simplifications introduced to reduce the complexity of the analysis while still preserving all the details relevant to analyze the structural design of the hall. The additional story atop the eastern second level rooms and the level beneath the western rooms are not represented. A uniform foundation at the ground level of the Great Hall is assumed for the entire model.

The entire Great Hall, with the exclusion of the travertine limestone supporting blocks at the impost of the vault, is built in opus caementicium with brick facing limited essentially to the vertical walls and the lower part of the piers. The concrete aggregate consists primarily of tuff fragments arranged in horizontal layers. To reduce the complexity of the modeling task, the entire structure, including the supporting blocks, is assumed to consist solely of opus caementicium, modeled, as indicated earlier, as a linear elastic material.

Three-dimensional FEM meshes are derived from the solid model comprising either the entire structural skeleton or a representative modular section. Typical interior and exterior sections are shown in Fig. 2 (a) and (b), respectively. The meshing and post-processing are done in ABAQUS CAE. Quadratic tetrahedral elements are used throughout the analyses. Appropriate kinematic boundary conditions are applied to the cut surfaces of the modular section meshes to simulate symmetry conditions. Linear elastic analysis is performed in ABAQUS Standard.

Displacements and Stress Results

The displacement results for the interior modular sections (Fig. 3) indicate that the deformation mechanism of the main vault consists of a vertical downward translation due to the compression exerted on the transverse walls and a flexural (bending) deformation with characteristic sagging of the extrados and maximum displacement at A. The side of the vault (line BC) undergoes a rotation with C moving inward and B outward. The lateral arches are bent downward without any appreciable change of length along the axis. The maximum deformation (3 mm at A) is well within the assumptions of infinitesimal strains.

As expected from the flexural deformation of the vault, the computed stress distribution indicates the presence of maximum principal tensile stresses in the x direction at the intrados, reaching pick value of 0.25 MPa at the crown, Fig. 4. Nuclei of equally elevated tensile stresses, also due to bending, are present at the attachments of the lateral arches. Maximum compressive stresses are almost everywhere below 0.6 MPa, with local peaks of 1.0 MPa. Similar displacement and stress results and peak values are obtained for the external modular section.

In order to evaluate separately the structural contribution of the contrasting arches, the supporting blocks, and the transverse walls, the interior modular section is modified either by removing the appropriate structural element (arch or wall) or by introducing sliding boundary conditions (between the blocks), and re-analyzed under gravitational loads. The removal of the contrasting arch does not affect the displacements and stresses in the vault, as shown for the tensile stresses at the intrados in Fig. 4. The suppression of the transverse wall, on the contrary, causes a dramatic increase in both deformations (Fig. 5) and tensile stresses in the vault as well as in the pier, which is now allowed to bend conspicuously. Similarly, sliding of the supporting blocks causes a marked increase of deformations (Fig 6) and tensile stresses at the intrados. Stresses are summarized in Tab. 1.

Critical Analysis of the Static Configuration

The results indicate that the lateral arches do not perform any appreciable contrasting function in resisting the horizontal thrust produced by the vault under static conditions. Rather, the necessary stabilizing action is provided by the transverse walls acting as shear resisting elements. These walls, however, terminate below the interface between the supporting blocks. The low setting of the shear walls together with the non-functionality of the contrasting arches forces the travertine blocks to assume a critical – and likely unintended – mechanical role. In addition to the vertical load, the entire horizontal thrust necessary to contain the bending of the vault must travel through the travertine blocks. Thus, sliding and rotation of the blocks, due to seismic loading or dif-
ferential foundation setting, becomes possible, since it cannot be prevented by the shear walls. This event would cause the stresses at the vault intrados to grow well beyond the tensile strength of the opus caementicium, giving rise to a longitudinal crack at the crown and possibly triggering a collapse mechanism. Indeed, physical evidence of extensive longitudinal fractures at the crown of the main vault, of substantial structural damage and repair on the lateral arches, and of the insertion of exterior metal clamps between each pair of travertine blocks suggests that near collapse conditions must have been reached during the early life of the structure (Leone; Margiotta 2007, p. 63), (Giuliani 2006, pp.267-268).

<table>
<thead>
<tr>
<th>Table 1: Maximum principal tensile stress at intrados</th>
<th>Great Hall [MPa]</th>
<th>Frigidarium [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal configuration</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Arch removed</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>Shear wall removed</td>
<td>0.75</td>
<td>2.20</td>
</tr>
<tr>
<td>Sliding of blocks allowed</td>
<td>0.65</td>
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**STRUCTURAL ANALYSIS OF THE FRIGIDARIUM**

**Geometric Modeling and FEM Meshing**

A detailed modern survey of the Frigidarium is not available and therefore the solid model of the Frigidarium, shown in Figs 1 and 7, is constructed by integrating data from several sources - primarily (Giovannoni 1925), (DeLaine 1997), (Cassanelli et al. 2002) and (Micozzi 2005) - with measurements taken by the authors. The structural skeleton consists of the main vault articulated into three cross vaults, the central one slightly larger than the other two. As for the Great Hall, the vault is supported by a system of blocks, transverse walls and constraining arches, Fig. 7. Each block, however, is fully incased in the transverse wall and, instead of projecting out as a true corbel, is carried by a monolithic column adjacent to the wall. The constraining arch is now a triangular buttressing element, mounted on top of the transverse wall and connected to the side of the vault in correspondence of the impost. The entire structure, with the exclusion of the columns and, presumably, the supporting blocks, is built in opus caementicium. Brick facing is limited to the vertical walls. Various types of brick ribbings and of aggregate are present in other vaults of the bath complex (Lancaster 2005) but the composition of the opus caementicium in the vault of the frigidarium is presently not visible. As for the model of the Great Hall, the present analysis assumes that the entire structure, including the blocks and column system, consists of opus caementicium represented as a linear elastic material.

![Figure 5: Deformations due to static gravitational loading without shear wall (left) or without arch (right)](image)

![Figure 6: Deformations due to static gravitational loading with fixed blocks (left) and with sliding blocks (right)](image)
The three-dimensional FEM model is limited to half of the central bay (Fig. 7). As before, appropriate boundary conditions are used to simulate structural symmetry along the cut surfaces. The analyses, based on quadratic tetrahedral elements are performed on ABAQUS Standard.

**Displacements and Stress Results**

The deformation of the vault under gravitational loading consists of a downward translation accompanied by bending both in the transverse and in the longitudinal plane, the latter due to the large span of the cross vault in the longitudinal (x) direction (Fig. 8). The maximum deformation is 8 mm, well within the constraints of the linear model. Due to biaxial bending, nuclei of nearly identical tensile stresses develop in the vault at the crown of the intrados and of the extrados with maximum value of 0.25 MPa (Fig. 8). As for the Great Hall, the modular section is modified by first removing the contrasting arches and then replacing the transverse walls with piers. The stress distribution due to gravitational loading is only moderately affected by the removal of the contrasting arch. The tensile stress pattern on the intrados and extrados remain the same, while the peak value grows by 20% (Fig. 9). The removal of the transverse wall, however, completely alters the stress picture. Now bending on the transverse plane becomes dominant and results in the tensile region spreading over large portions of the intrados with peaks of 2.2 MPa (Fig. 9). As for the Great Hall, high tensile stresses develop in the piers due to bending. Peak stress values for the different configurations are summarized in Tab. 1.

**Critical Analysis of the Static Configuration**

As configured in the Frigidarium, the lateral arch and the transverse walls form a structural continuum, which we consider separately to allow for a direct comparison with the Great Hall. The results indicate that the lateral arches perform only a minor contrasting function in resisting the horizontal thrust of the vault, while, as for Great Hall, the critical stabilizing action is produced by the transverse walls. Most importantly, because the supporting blocks are now fully encased in the *opus caementicium* of these walls, there is no the possibility of horizontal sliding at the points of support of the vault. The weakest link in the structural organism of the vault, identified in the analysis of the Great Hall, has been effectively eliminated.
CONCLUSIONS: EVOLUTION OF THE CROSS VAULT DESIGN

The design choice for the Great Hall, consisting of low shear walls and high contrasting arches, with interposed travertine blocks, is at best problematic. Friction between the travertine blocks - and, equally, adhesion betweenopus caementicium and travertine - becomes the physical parameter controlling the stability of the configuration, arguably a poor choice, since these surfaces must transmit a sizeable horizontal force from the impost of the vault to the shear wall. Also, the lateral arch, due to its high position, is structurally ineffectual. Yet, as noted in (Giovannoni 1913), the shear wall is correctly positioned and properly sized. In fact, as long as there is no motion between the blocks, the wall provides the appropriate reaction necessary to control the deformation of the vault.

The structural configuration of the Great Hall cannot be applied to a much larger building such as the Frigidarium without catastrophic consequences. FEM analysis shows that scaling the Great Hall model so that it matches the span of the Frigidarium leads to tensile stresses at the intrados far exceeding the strength limit of the material. Conversely, redesigning the vault support system of the Great Hall according to the structural scheme of the Frigidarium lowers the tensile stresses at the crown to less than half of the original value! The analysis of the Frigidarium shows that the structural deficiencies revealed in the analysis of Great Hall have been systematically corrected: the shear wall is extended upward, the contrasting arch is lowered and becomes an integral part of the shear wall, and the supporting blocks are completely encased in the opus caementicium. In addition, the projecting blocks are now supported by columns. It is important to realize that, based on surviving physical evidence, the cross vault design of the Great Hall is unique, while the configuration of Diocletian’s Frigidarium is essentially identical to the structural design of the Frigidarium of Caracalla’s Baths (A.D. 212 – 216) and of the Basilica of Maxentius (A.D. 307-315), the two other gigantic halls still partially standing in Rome. This, and the fact that, as we indicated earlier, the Great Hall shows clear and multiple evidence of structural damage, lead us to assume that Roman engineers detected and correctly interpreted the structural deficiencies of the Great Hall and, based on the acquired knowledge, proceeded to develop new design paradigms for cross-vaulted halls, which allowed them to successfully build some of the largest masonry vaults in history.

How should we interpret the design of the Great Hall? Again taking into account that its structural configuration is unique, two scenarios are possible. Either the Great Hall is the first attempt to design a free standing cross vault in opus caementicum of sizeable dimensions, or it represents a deviation from an already established design paradigm – a deviation whose peculiarities where dictated by considerations external to structural engineering. The latter scenario seems unlikely, since it implies that the designer intentionally selected a configuration inherently weak and potentially catastrophic, making a technical choice which would be difficult to reconcile with the importance of the project and the caliber of the technical staff that must have been in charge of it. Simply stated, the choice of an inferior structural solution is incompatible with the imperial nature of the project. On the contrary, the very notion of a prestigious commission that required a new technical solution and resulted in the creation of a new structural form – the cross vault – can help explain the innovative aspects and the unexpected deficiencies of the design the Great Hall. Therefore, it is reasonable to assume that the structural designer was unaware of the mechanical implications of the design choice.

As for the design intention, the configuration of the supports and the positioning of the contrasting arches suggest the following consideration. The support system was designed to prevent an outward rotation hinged on the travertine blocks. In this case, the arch is positioned correctly, as distant as possible from the rotation center – the blocks - in order to provide the maximum restoring effect without having to develop an excessive compressive force. With the arch preventing the rotation, the load carried by the travertine blocks is predominantly vertical and therefore sliding of the blocks can be safely ignored. The notion that a vaulted structure could collapse through outward rotation is consistent with the collapse mechanism of an arched structure built with stone blocks (opus quadratum), a type of failure certainly known to structural designers in Trajanic times. Therefore the design intention can be interpreted as the creation of a modular structural form, derived from intersecting arched configurations and to be constructed in opus caementicium, but conceived to behave - and thus fail - like a masonry block structure. The weakness of the design reflects the ignorance of material behavior. Thus the Great Hall resulted from an engineering practice that had yet to acquire the correct me-
chancial understanding of how an arched structure in opus caementicium fails. Perhaps, its near structural failure provided the stimulus and the necessary visible evidence for developing this knowledge.

Finally, we return on the applicability of linear FEM to concrete. As discussed in (Samuelli Ferretti 2005), linear elastic FEM models of concrete provide acceptable results only as long as the computed stresses remain within the elastic range. In the present study, as shown in Table 1, this is indeed the case for both principal stresses in tension and compression computed for the normal configuration of the monuments. Principal tensile stresses exceed the tensile strength only when we alter the normal configuration by removing a critical structural element. We do so, however, only to illustrate the catastrophic effect induced by such a change, and not to provide an accurate evaluation of the stresses in the modified configuration.

REFERENCES


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